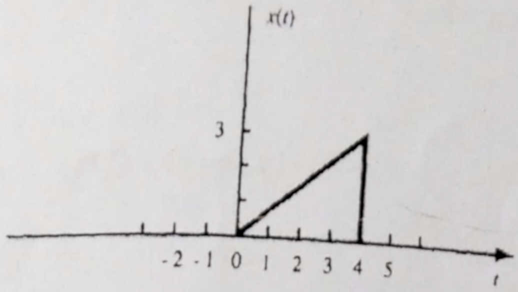
كلية التقنية الالكترونية/طرابلس

Signal & Systems

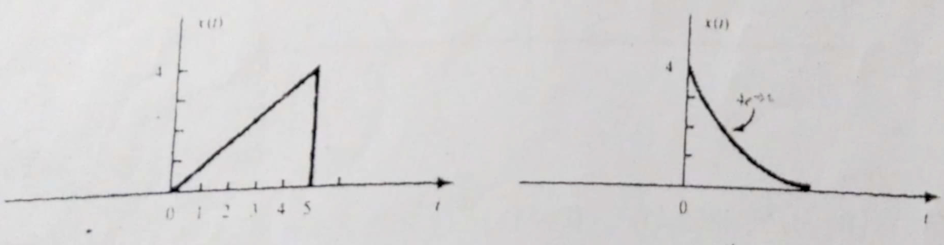
Sheet #1

Q-1 A continuous-time signal **x (t)** is shown in Fig. Sketch and label each of the following signals.

a- $x(t-2)$ b- $x(2t)$ c- $x(\frac{t}{2})$ d- $x(-t)$



Q-2 *Sketch and label the even and odd components of the signals shown in Fig.*



(a) (b)

Q-3 Show that the complex exponential signal $x(t) = e^{j\omega_o t}$ is periodic and that its fundamental period is $T = \frac{2\pi}{\omega_o}$

Q-4 Show that the sinusoidal signal $x(t) = \cos(\omega_o t + \theta)$ is periodic and that its fundamental period is $\frac{2\pi}{\omega_o}$

Q-5 *Determine whether or not each of the following signals is periodic. If a signal is periodic, determine its fundamental period.*

a) $x(t) = \cos(t + \frac{\pi}{4})$ *b)* $x(t) = \sin(\frac{2\pi}{3})$

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c) $x(t) = \cos\frac{\pi}{3}t + \sin\frac{\pi}{4}t$

d) $x(t) = \sin^2 t$

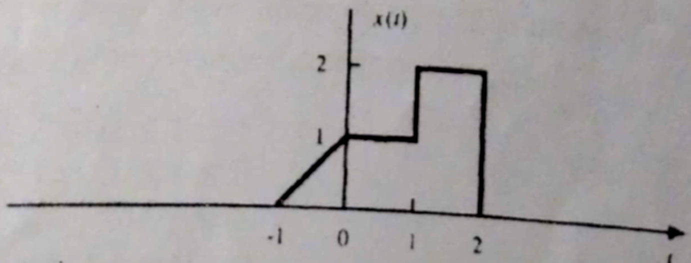
e) $x(t) = \cos t + \sin\sqrt{2}\,t$

Q-6 Determine whether the following signals are energy signals, power signals, or neither.

(a) $x(t) = e^{-at}$, $a > 0$ b)- $x(t) = A\cos(\omega t + \theta)$ c) $x(t) = tu(t)$

Q-7 A continuous-time signal x(t) is shown in Fig. Sketch and label each of the following signals.

(a) $x(t)u(1-t)$; (b) $x(t)[u(t)-u(t-1)]$; (c) $x(t)\delta(t-\frac{1}{2})$



Q-8 Show that

(a) $t\delta(t) = 0$ (b) $\sin(t)\,\delta(t) = 0$ (c) $\cos(t)\delta(t-\pi) = -\delta(t-\pi)$

Q-9 Evaluate the following integrals:

a) $\int_{-1}^{1}(3t^2+1)\,\delta(t)dt$ b) $\int_{1}^{2}(3t^2+1)\delta(t)dt$

b) $\int_{-\infty}^{\infty}(t^2+\cos(\pi t))\,\delta(t-1)dt$ d) $\int_{-\infty}^{\infty} e^{-t}\,\delta(t-2)dt$

e) $\int_{-\infty}^{\infty} e^{-t}\delta(t)dt$

Q-10 Find and sketch the first derivatives of the following signals:

a) $x(t) = u(t) - u(t-a)$, $a > 0$

b) $x(t) = t[u(t) - u(t-a)]$, $a > 0$

c) $x(t) = u(t+2) + u(t) - 2u(t-3)$

Q- 11 Find the even and odd components of the signal $x(t) = e^{-2t}\cos(t)$

Q-12 Find the even and odd components of each of the following signals:

$a - x(t) = \cos(t) + \sin(t) + \sin(t)\cos(t)$

b- $x(t) = 1 + t + 3t^2 + 5t^3 + 9t^4$

c- $x(t) = 1 + t\cos(t) + t^2\sin(t) + t^3\sin(t)\cos(t)$

d- $x(t) = (1 + t^3)\cos^3(10t)$

Q-13 For each of the following signals, determine whether it is periodic, and if it is find the fundamental period:

a- $x(t) = \cos^2(2\pi t)$

b- $x(t) = \sin^2(2t)$

c- $x(t) = e^{-2t}\cos(2\pi t)$

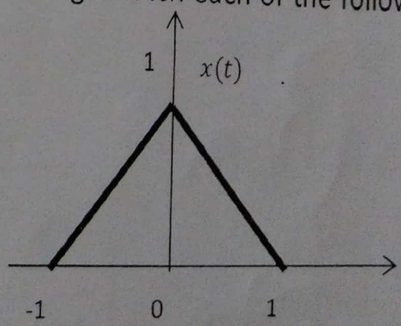
Q-14 Categorize each of the following signals as an energy signal or a power signal and find the energy or time averaged power of the signal:

a- $x(t) = \begin{cases} t, & 0 \le t \le 1 \\ 2 - t, & 1 \le t \le 2 \\ 0, & otherwise \end{cases}$

b- $x(t) = 5\cos(\pi t) + \sin(5\pi t), \quad -\infty < t < \infty$

c- $x(t) = f(x) = \begin{cases} 5\cos(\pi t), & -1 < t < 1 \\ 0, & otherwise \end{cases}$

Q-15 A triangular pule signal x(t)is depicted in fig. Sketch each of the following signals derived from x(t)

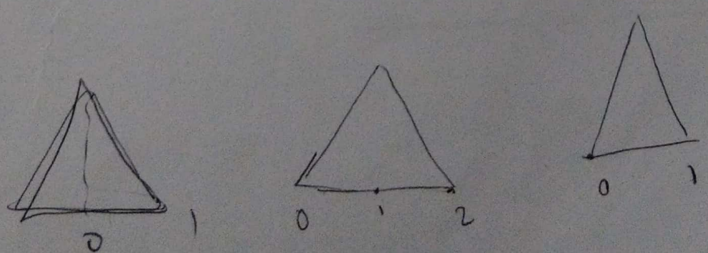


a- $x(3t)$ b) $x(3t + 2)$

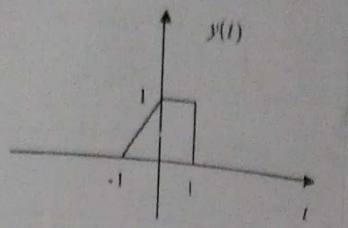
c) $x(-2t - 1)$ d) $x(2(t + 2))$

e) $x(2(t - 2))$ f) $x(3t) + x(3t + 2)$

g) $x(t) = 2 + x(3t + 2)$



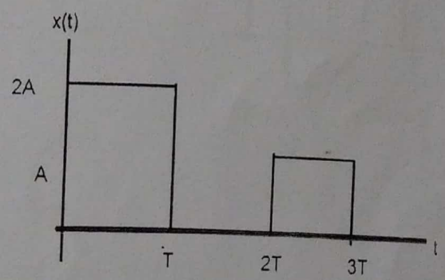
Q-16 ***Find the even and odd components of the signal*** $y(t)$



Q-17 Sketch the signal:

$$f(t) = -u(-t) + u(t) + [2u(t+2) - 2u(t-2)$$

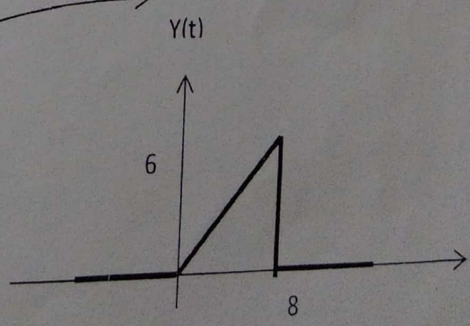
Q-18 For the signal showen and using the definetion of the unit step function. Write the expression of: $a)\ x(t)$ $\qquad b)\ \frac{dx(t)}{dt}$



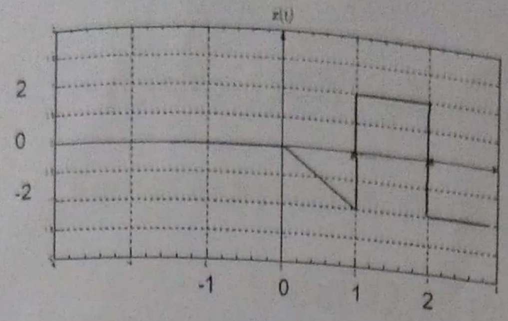
Q-19 Evaluate the value of:

a. $x(t) = [\frac{2+jt}{3+jt^2}].\delta(t)$

b. $f(t) = \int_{-\infty}^{\infty} y(t)\,\delta(t-2)$



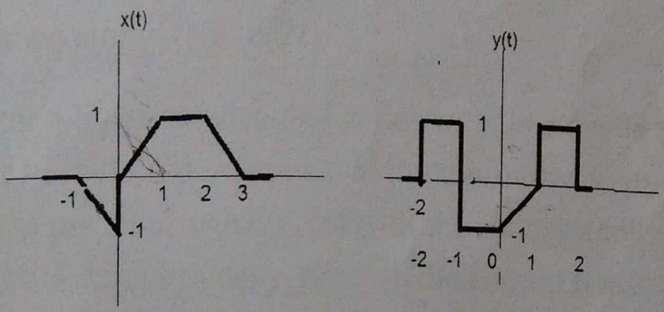
Q-20 Write the expression of the following signal:



Q-21 For the two signals $x(t)$ *and* $y(t)$ Carefully sketch the following signals:

a. $x(t)y(t-1)$ b. $x(t-1)y(-t)$ c. $x(t+1)y(t-2)$

b. $x(t)y(2-t)$ e. $x(t)y(-1-t)$ f. $x(2t)y(\frac{1}{2}t+1)$



g. $x(4-t)y(t)$ h. $x(t)+y(t)$ i. $x(t)-y(t)$

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Signal & Systems part(2)

Sheet #2

Q1) Determine whether the system $y(t) = x(t)\cos(\omega_c t)$ is(a) memoryless (b)causal (c) stable (d) linear or (e) time invariant.

Q2) Consider the system with the input-output relation $y(t) = x(t) + 1$

Classify this system.

Q3) Classify the system $y(t) = x(2t)$

Q5) A thermistor has a resistance that varies with time due to temperature change; let $R(t)$ denote the resistance of the thermistor, expressed as function of time. Associating the input signal $x_1(t)$ with the voltage applied across the thermistor and the output signal $y_1(t)$ with the current flowing through the thermistor, we may express the input- output relation of the device as:

$$y_1(t) = \frac{x_1(t)}{R(t)}.$$

Show that the thermistor described is time variant.

Q4) Consider a system described by the input- output relation

$$y(t) = x(t)x(t-1)$$

Show that the system is nonlinear.

Q5) The systems that follow have input $x(t)$ and output $y(t)$,For each system, determine whether it is (i) memoryless; (ii) stable; (iii) causal; (iv) linear and (v) time invariant.

(a) $y(t) = \cos(x(t))$
(b) $y(t) = x(2-t)$

(c) $y(t) = x(\frac{t}{2})$

Q(5) Consider the CT systems with the following input- output relations:

a) $y(t) = \frac{dx(t)}{dt}$

b) $y(t) = e^{x(t)}$

c) $y(t) = 3x(t) + 5$

Determine whether the CT systems are linear.

Q(6) Consider the AM system with the input- output relation given by:

$$s(t) = [1 + 2m(t)]\cos(2\pi \times 10^8 t)$$

Determine if the AM system is linear.

Q(7) Consider two CT systems represented by the following input- output relationship:

I. System1 $\quad y(t) = \sin(x(t))$

II. System2 $\quad y(t) = t\sin(x(t))$

Determine if system1 and system2 are time invariant.

Q(8) Determine if the following systems are linear, time invariant, stable, causal and memoryless:

a) $y(t) = x(t-2)$

b) $y(t) = x(2t-5)$

c) $y(t) = tx(t+10)$

d) $y(t) = \begin{cases} 2 & x(t) \geq 0 \\ 0 & x(t) < 0 \end{cases}$

e) $y(t) = \begin{cases} 0 & t < 0 \\ x(t) - x(t-5) & t \geq 0 \end{cases}$

f) $y(t) = 7x^2(t) + 5x(t) + 3$

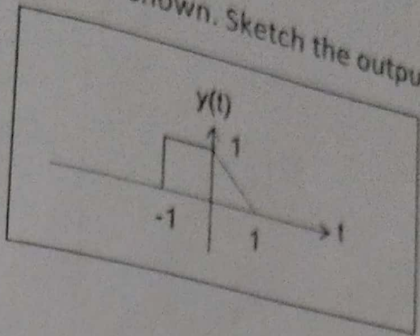
g) $y(t) = sgn(x(t))$

$tx(t+x(t))+5$

① $y(t) = tx(t+x(t-t_0)+5$

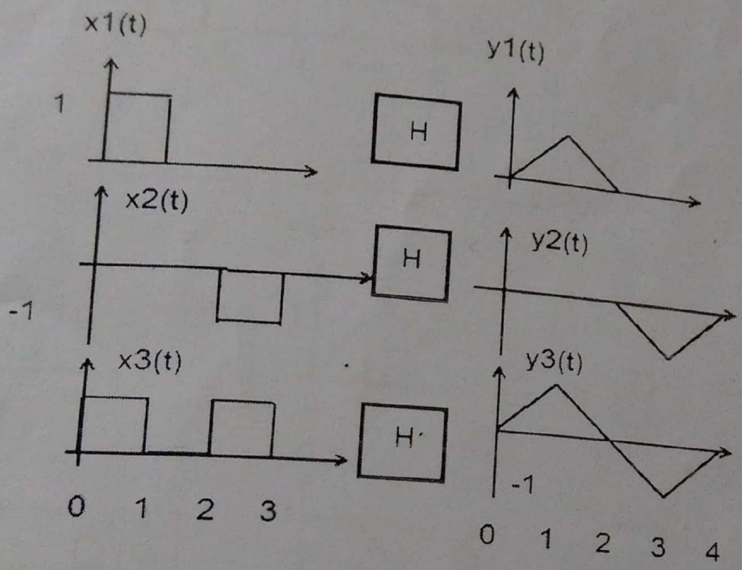
② $y(t-t_0)\ x(t-t_0-x(t-t_0))+5$

Q(9) For an LTI system, an input $x(t)$ produces $y(t)$ as shown. Sketch the output for the following set of inputs:



a) $5x(t)$
b) $0.5x(t-1)+0.5x(t+1)$
c) $x(t+1)-x(t-1)$
d) $\frac{dx(t)}{dt}+3x(t)$

Q(10) A system (H) has its input- output pairs given. Determine whether the system could be memoryless, causal, linear, and time invariant.



Q11) Repeat (Q10) for the given input- output pairs [Figure (Q11).

Q12) A linear system (H) has the input- output pairs as shown,. Answer the following questions, and explain your answer:

a) Could the system be causal?

b) Could this system be time invariant?

c) Could the system be memoryless?

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d) What is the output for the input depicted in (b)?

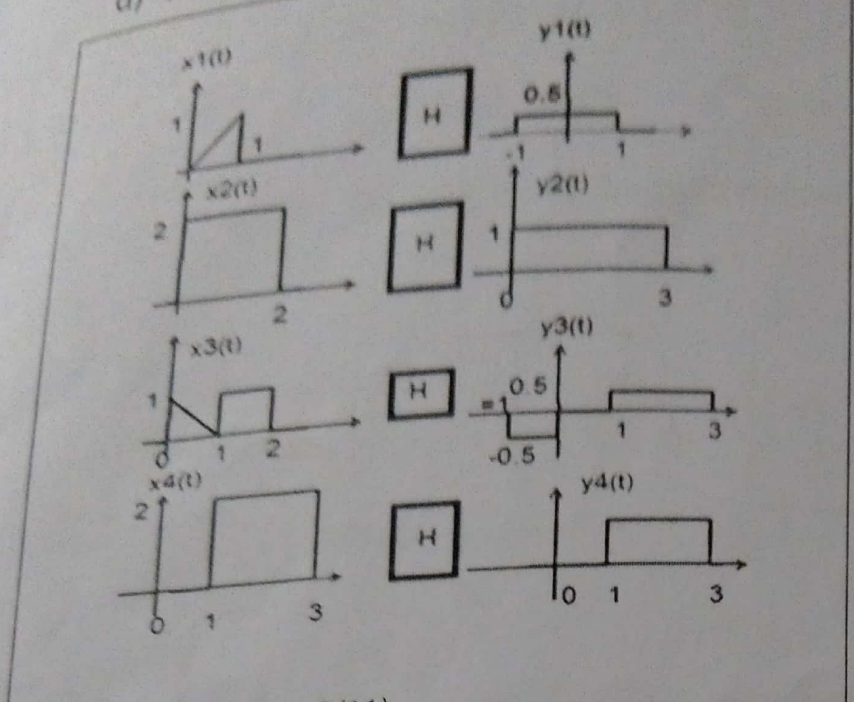


Figure Q(11)

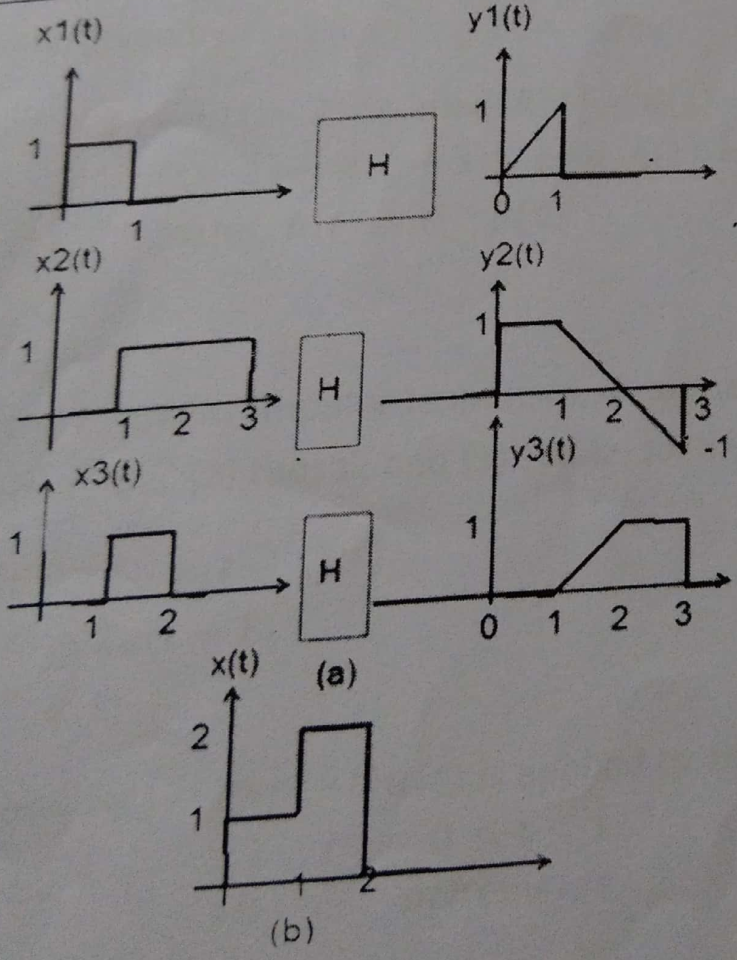
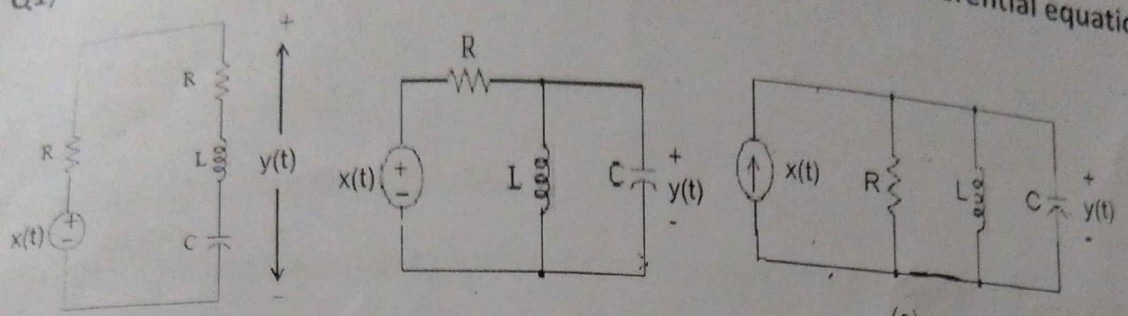


Figure Q(12)

Signal & Systems part3

Sheet #3

Q1) For the shown circuits. Find the input- output relation as a differential equation.



(a) (b) (c)

Q2) Evaluate the convolution integral for a system with input $x(t)$and impulse response $h(t)$, respectively, given by:

a) $x(t) = u(t-1) - u(t-3)$ and $h(t) = u(t) - u(t-2)$
b) $x(t) = (t-1)[u(t-1) - u(t-3)]$ and $h(t) = u(t+1) - 2u(t-2)$
c) $x(t) = u(t)$ and $h(t) = e^{-t}\, u(t)$

Q3) For each of the following impulse responses, determine whether the corresponding system is (1) memoryless, (2) causal, and (3) stable. Justify your answers.

a) $h(t) = u(t+1) - u(t-1)$
b) $h(t) = u(t) - 2u(t-1)$
c) $h(t) = e^{-|t|}\, u(t)$

Q4) The input signal $x(t) = exp(-t)u(t)$ is applied to an LTI system whose impulse response is given by: $h(t) = \begin{cases} 1-t & 0 \le t \le 1 \\ 0 & otherwise \end{cases}$

Q5) Calculate the output for the following input signal and impulse response:

$x(t) = \begin{cases} 1.5 & -2 \le t \le 3 \\ 0 & otherwise \end{cases}$ and $h(t) = \begin{cases} 2 & -1 \le t \le 2 \\ 0 & otherwise \end{cases}$

Q6) Determine if the systems with the following impulse response:

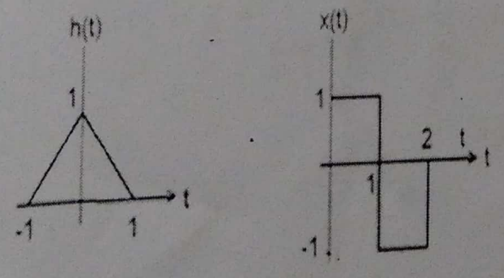
I. $h(t) = \delta(t) - \delta(t-2)$
II. $h(t) = 2rect(\frac{t}{2})$
III. $h(t) = 2exp(-4t)u(t)$
IV. H(t)=[1-exp(-4t)]u(t)

Are memoryless, causal, and stable.

Q7) Determine the output $y(t)$ for the following pairs of input signal $x(t)$ and impulse response $h(t)$:

I. $x(t) = u(t)$, $h(t) = u(t)$
II. $x(t) = u(-t)$, $h(t) = u(-t)$
III. $x(t) = u(t) - 2u(t-1) + u(t-2)$, $h(t) = u(t+1) - u(t-1)$

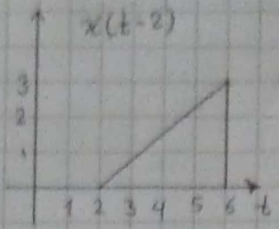
Q8) The input and response of LTI system are as shown, determine the output.



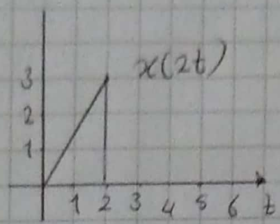
Sheet #1

Q1

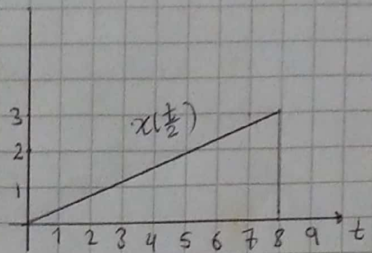
a) $x(t-2)$



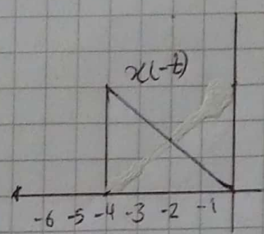
b) $x(2t)$



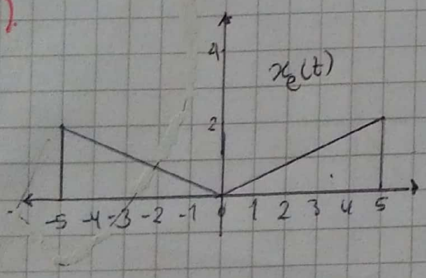
c) $x(\frac{t}{2})$



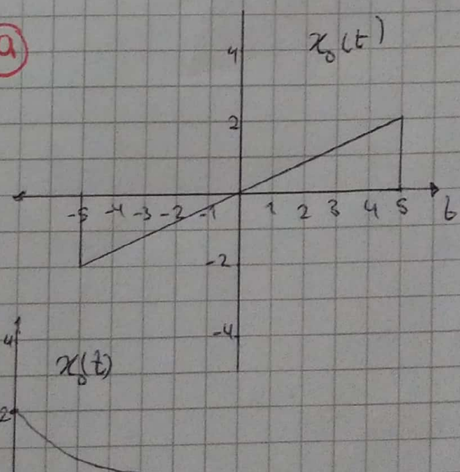
d) $x(-t)$

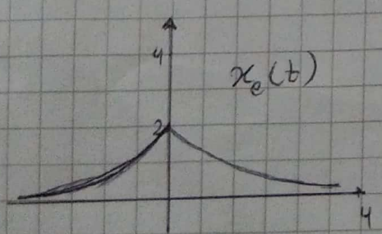


Q2)



a





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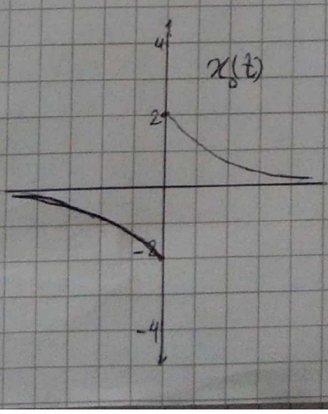

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Q3:-

$x(t) = e^{j\omega_0 t}$ is periodic iif $e^{j\omega_0 t} = e^{j\omega_0 (t+T_0)}$

$e^{j\omega_0 t} = e^{j\omega_0 t} . e^{j\omega_0 T_0}$

For the two sides to be equal $\Rightarrow e^{j\omega_0 T_0} = 1$

$e^{j\omega_0 T} = \cos(\omega_0 T_0) + j\sin(\omega_0 T_0) = 1 + j(0)$

$\cos(\omega_0 T_0) = 1 \Rightarrow \omega_0 T_0 = \cos^{-1}(1)$

$\omega_0 T_0 = 2\pi$

$T_0 = \frac{2\pi}{\omega_0}$ #

Q4:- for $\cos(\omega_0 t + \theta)$ to be periodic ~~then~~

$\cos(\omega_0 t + \theta) = \cos(\omega_0 (t + T_0) + \theta)$

$\cos(a + b) = \cos(a)\cos(b) - \sin(a)\sin(b)$

$\cos(\omega_0 t + \theta) = \cos(\underbrace{\omega_0 t + \theta} + \underbrace{\omega_0 T_0})$

$\cos(\omega_0 t + \theta) = \cos(\omega_0 t + \theta)\cos(\omega_0 T_0) - \underbrace{\cancel{\sin(\omega_0 t + \theta)}\sin(\omega_0 T_0)}_{=0}$

$\cos(\omega_0 t + \theta) = \cos(\omega_0 t + \theta)\cos(\omega_0 T_0)$

$\cos(\omega_0 T_0) = 1 \Rightarrow \omega_0 T_0 = \cos^{-1}(1)$

$\omega_0 T_0 = n 2\pi \Rightarrow$ where n is integer, $0, \pm1, \pm2, \pm3, \ldots$

for $n = 1 \Rightarrow \omega_0 T_0 = 2\pi$

$T_0 = \frac{2\pi}{\omega_0}$ #

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Q6:

a) $\cos(t+\frac{\pi}{4})$ Periodic on form $\cos(\omega_0 t+\frac{\pi}{4})$

$\omega_0 = 1 \rightarrow T_0 = \frac{2\pi}{1} = 2\pi$ (sec)

b) $x(t) = \sin(\frac{2\pi}{3}t) \Rightarrow \omega_1 = \frac{2\pi}{3}$

periodic with $T_0 = \frac{2\pi}{\frac{2\pi}{3}} = 3$ (sec)

c) $x(t) = \cos(\frac{\pi}{3}t) + \sin(\frac{\pi}{4}t)$

$x_1(t) = \cos(\frac{\pi}{3}t) \Rightarrow$ periodic with $\omega_1 = \frac{\pi}{3}$

and $T_1 = \frac{2\cancel{\pi}}{\frac{\cancel{\pi}}{3}} = 6$ (sec)

$x_2(t) = \sin(\frac{\pi}{4}t) \Rightarrow$ periodic with $\omega_2 = \frac{\pi}{4}$

and $T_1 = \frac{2\cancel{\pi}}{\frac{\cancel{\pi}}{4}} = 8$ (sec)

$\frac{T_1}{T_2} = \frac{6}{8}$, is a rational number

Then $x(t)$ is periodic with a fundamental

period $T_0 = 24$ (sec)

d) $x(t) = \sin^2 t$

$x(t) = \frac{1-\cos(2t)}{2} \Rightarrow \frac{1}{2} - \frac{1}{2}\cos(2t)$

The constant $(\frac{1}{2})$ is an increase in the amplitude and does not affect the periodicity.

Date:

$-\frac{1}{2}\cos(2t)$ is periodic with $\omega = 2$

$T = \frac{2\pi}{2} = \pi$ (sec)

Then $x(t)$ is periodic with $T_0 = (\pi)$ sec

(e) $x(t) = \cos(t) + \sin(\sqrt{2}t)$

$x_1(t) = \cos(t)$ is periodic with $\omega_1 = 1$

$T_1 = 2\pi$ (sec)

$x_2(t)$ is periodic $\Rightarrow x_2(t) = \sin(\sqrt{2}t)$ $\omega_2 = \sqrt{2}$

$T_2 = \frac{2\pi}{\sqrt{2}} = \sqrt{2}\pi$ (sec)

$\frac{T_1}{T_2} = \frac{2\cancel{\pi}}{\sqrt{2}\cancel{\pi}} = \sqrt{2}$ which is irrational number.

$x(t)$ is not periodic.

Q6)

(a) $x(t) = e^{-at} u(t)\, a > 0$

$$E = \int_{-\infty}^{\infty} |x(t)|^2 . dt = \int_{0}^{\infty} (e^{-at})^2 . dt$$

$$E = \int_{0}^{\infty} e^{-2at} . dt \Rightarrow E = \frac{1}{-2a} e^{-2at} \Big|_{0}^{\infty}$$

$$E = \frac{1}{2a} \left[e^{-2a(0)} - e^{-2a(\infty)} \right] = \frac{1}{2a} \left[e^{0} - \cancel{e^{-\infty}}^{=0} \right]$$

$= \frac{1}{2a}$ Joule. $\frac{1}{2a} < \infty$ so it is energy signal.

(b) $x(t) = A \cos(\omega t + \theta)$

The signal is periodic, so it is power signal

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} |x(t)| . dt$$

and for periodic signal:-

$$P = \frac{1}{T_0} \int_{0}^{T_0} |x(t)|^2 . dt$$

$$T_0 = \frac{2\pi}{\omega_0} \Rightarrow P = \frac{\omega_0}{2\pi} \int_{0}^{\frac{2\pi}{\omega_0}} A^2 \cos^2(\omega_0 t + \theta)$$

$$P = \frac{\omega_0 A^2}{2\pi} \int_{0}^{\frac{2\pi}{\omega_0}} \frac{1}{2} \left(1 + \cos(2\omega_0 t + 2\theta)\right) . dt$$

$$P = \frac{A^2 \omega_0}{4\pi} \left[t + \frac{\sin(2\omega_0 t + 2\theta)}{2\omega_0} \right]_{0}^{\frac{2\pi}{\omega_0}}$$

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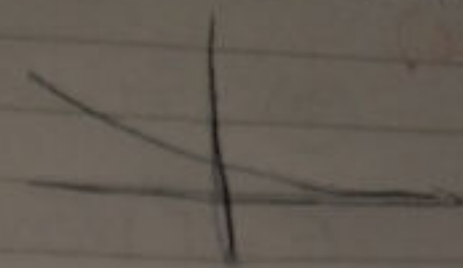
(6) (a) e^{-at}, $a>0$

$$E_t = \int_{-\infty}^{\infty} e^{-2at}\,dt$$

$$= -\frac{e^{-2at}}{2a}\Big|_{-\infty}^{\infty} = \frac{-e^{-2a(\infty)} + e^{-2a(-\infty)}}{2a} = \frac{0+\infty}{2a} = \infty$$



not energy signal.

$$P_t = \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} e^{-2at}\,dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \left[\frac{-e^{-2at}}{2a}\right]_{-\frac{T}{2}}^{\frac{T}{2}}$$

$$\Rightarrow \lim_{T\to\infty} \frac{1}{T} \left[\frac{-e^{-2a\frac{T}{2}} + e^{2a\frac{T}{2}}}{2a}\right] = \cancel{e^{-2}}$$

$$= \frac{1}{\infty}\left[\frac{e^{-2a\infty} + e^{2a\infty}}{2a}\right] \Rightarrow \infty$$

neither energy nor power signal.

$$P = \frac{A^2 \omega_0}{4\pi}\left[\frac{2\pi}{\omega_0} + \frac{\sin(2\omega_0 \cdot \frac{2\pi}{\omega_0} + 2\theta)}{2\omega_0} - \frac{\sin(2\theta)}{2\omega_0}\right]$$

$$P = \frac{A^2 \omega_0}{4\pi}\left[\frac{2\pi}{\omega_0} + \frac{\sin(2\theta + 4\pi) - \sin(2\theta)}{2\omega_0}\right]$$

* $\sin(\theta + 2n\pi) = \sin(\theta)$

$$P = \frac{A^2 \omega_0}{4\pi}\left[\frac{2\pi}{\omega_0} + \underbrace{\cancel{\frac{\sin(2\theta) - \sin(2\theta)}{2\omega_0}}}_{\text{zero}}\right]$$

$$P = \frac{A^2 \omega_0}{4\pi}\left(\frac{2\pi}{\omega_0}\right)$$

2

$$P = \frac{A^2}{2} \text{ watt.}$$

$P < \infty$ so it is power signal

* In general the power of sinusoidal signal in the form of $\Rightarrow x(t) = A(\cos \omega t + \theta)$ is equal to $\frac{A^2}{2}$ watt.

ⓒ $x(t) = t\,u(t)$

$$E = \int_{-\infty}^{\infty} |x(t)|^2 . dt = \int_0^{\infty} t^2 . dt = \frac{t^3}{3}\Big|_0^{\infty} = \infty$$

$$P = \lim_{T\to\infty} \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} |x(t)|^2 . dt = \lim_{T\to\infty} \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} t^2 . dt$$

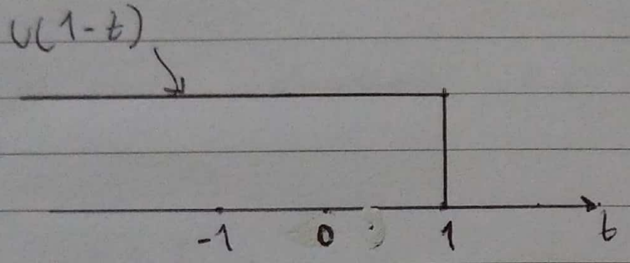
$$= \lim_{T\to\infty} \frac{1}{T} \frac{t^3}{3}\Big|_{-\frac{T}{2}}^{\frac{T}{2}} = \lim_{T\to\infty} \frac{1}{3T}\left[\frac{T^3}{8} + \frac{T^3}{8}\right]$$

$$P = \lim_{T \to \infty} \frac{1}{3\cancel{T}} * \frac{T^{\cancel{3}\,2}}{4}$$

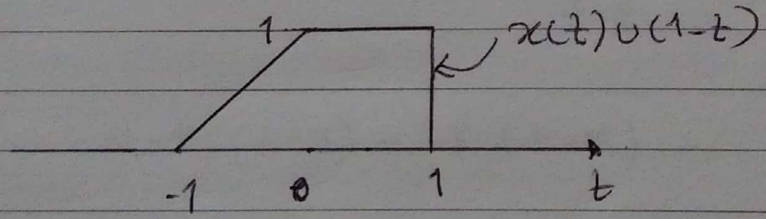
$$P = \lim_{T \to \infty} \left[\frac{T^2}{12}\right] = \infty$$

Therefore, $x(t)$ is neither an energy signal nor Power signal.

Q7: - ⓐ $x(t)\,u(1-t)$

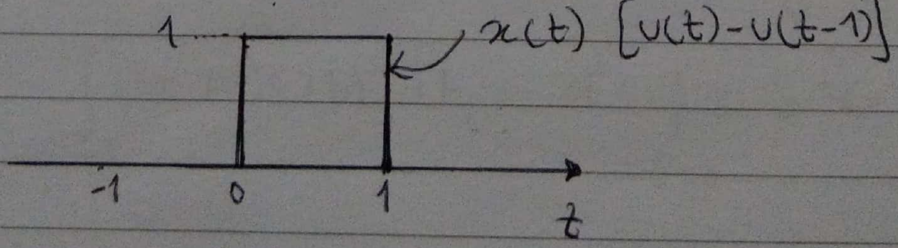


$x(t)\,u(1-t)$ will eleminate all signal parts after $t=1$,



ⓑ $x(t)\left[u(t)-u(t-1)\right]$

Only signal parts between $(0,1)$ will remain.



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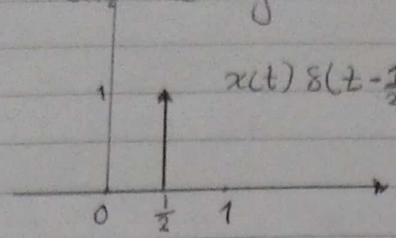
ⓒ $x(t)\,\delta(t-\frac{1}{2})$

This means that the signal will exist only at $t=\frac{1}{2}$



Q8:-

ⓐ $t\,\delta(t) = t\,\delta(t-0) = t\Big|_{t=0}\delta(t) = 0\,\delta(t) = 0$ #

ⓑ $\sin(t)\,\delta(t) = \sin(t)\,\delta(t-0) = \sin(t)\Big|_{t=0}\delta(t) = \sin(0)\,\delta(t)$

$= 0\,\delta(t) = 0$ #

ⓒ $\cos(t)\,\delta(t-\pi) = \cos(t)\Big|_{t=\pi}\delta(t-\pi) = \cos(\pi)\,\delta(t-\pi)$

$= -\cancel{1}\delta(t-\pi) = -\delta(t-\pi)$.

Q9:-

ⓐ $\int_{-1}^{1}(3t^2+1)\,\delta(t)\,dt$

$\delta(t-0) \Rightarrow 0$ is with in the integration limits

So:- $\int_{-1}^{1}(3t^2+1)\,\delta(t)\,dt = 3t^2+1\Big|_{t=0} = 3(0)+1 = 1$

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(b) $\int_{1}^{2}(3t^2+1)\,\delta(t)\,dt$ 0 is not within the integration limits

So $\int_{1}^{2}(3t^2+1)\delta(t)\,dt = 0$

(c) $\int_{-\infty}^{\infty}(t^2+\cos(\pi t))\,\delta(t-1)\,dt$

$\delta(t-1) \Rightarrow 1$ is within the integration limits

$$\int_{-\infty}^{\infty}(t^2+\cos(\pi t))\,\delta(t-1)\,dt = t^2+\cos(\pi t)\Big|_{t=1} = 1+\cos(\pi) = 0$$

(d) $\int_{-\infty}^{\infty} e^{-t}\,\delta(t-2)\,dt$, $t=2$ is within the limits of $\int$

$$\int_{-\infty}^{\infty} e^{-t}\,\delta(t-2)\,dt = e^{-t}\Big|_{t=2} = e^{-2} = \frac{1}{e^2} = 0.135$$

(e) $\int_{-\infty}^{\infty} e^{-t}\,\delta(t)\,dt \Rightarrow t=0$ is within $\int$ limits.

$$= e^{-t}\Big|_{t=0} = e^{0} = 1$$

Q10:- (a) $\frac{dx}{dt} = \delta(t) - \delta(t-1)$

(b) $\frac{dx}{dt} = t\left[\delta(t)-\delta(t-a)\right] + 1\left[u(t)-u(t-a)\right]$

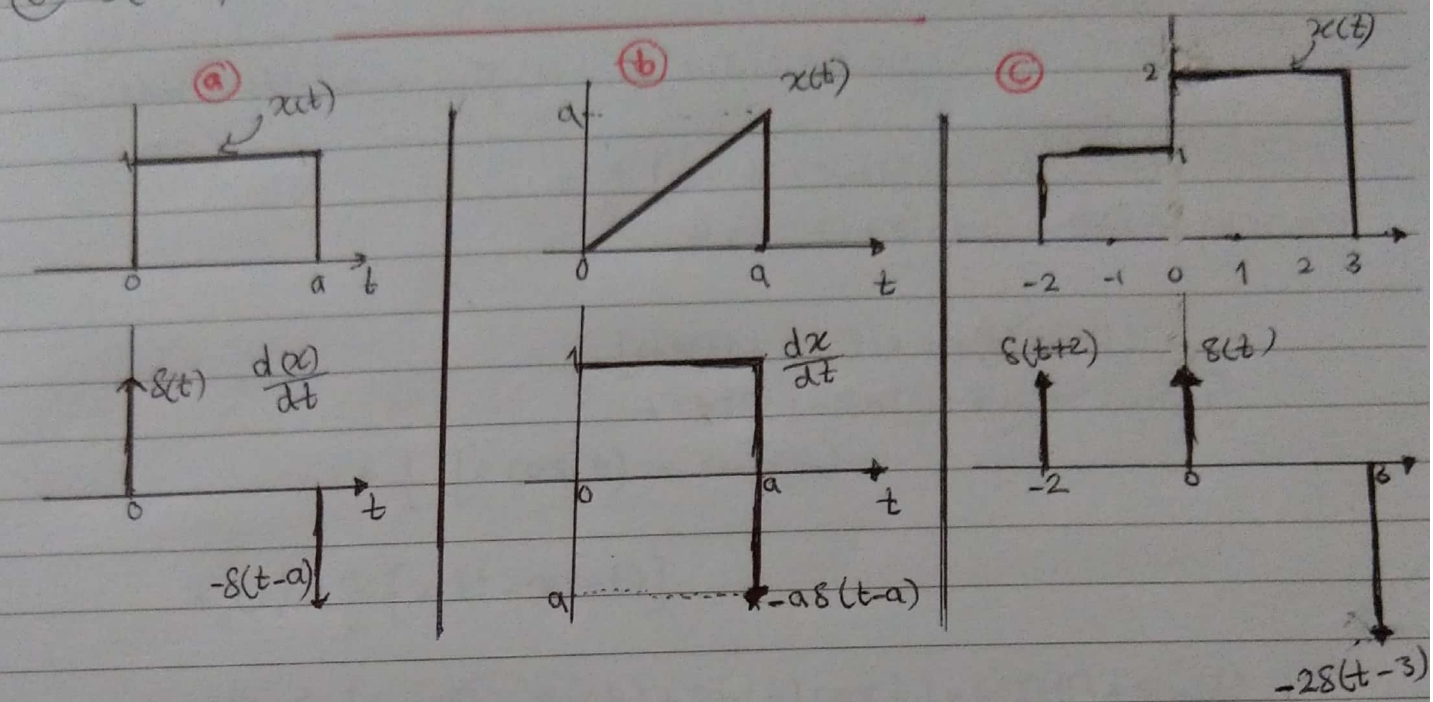
$= t\,\delta(t) - t\,\delta(t-a) + u(t) - u(t-a)$

$= t\big|_{t=0}\,\delta(t) - t\big|_{t=a}\,\delta(t-a) + u(t) - u(t-a)$

$= 0 - a\delta(t-a) + u(t) - u(t-a)$

$= u(t) - u(t-a) - a\delta(t-a)$

(c) $\delta(t+2) + \delta(t) - 2\delta(t-3)$



Q.11 :- $x_e(t) = \frac{1}{2}\left[x(t) + x(-t)\right]$

$= \frac{1}{2}\left[e^{-2t}\cos(t) + e^{-(-2t)}\cos(-t)\right]$

$= \frac{1}{2}\left[e^{-2t}\cos(t) + e^{2t}\cos(t)\right]$

$= \frac{1}{2}\cos(t)\left[e^{-2t} + e^{2t}\right] \Rightarrow \frac{e^{2t} + e^{-2t}}{2} = \cosh(2t)$

$x_o(t) = \frac{1}{2}\left[x(t) - x(-t)\right]$

$= \frac{1}{2}\left[e^{-2t}\cos(t) - e^{-(-2t)}\cos(-t)\right]$

$= \frac{1}{2}\left[e^{-2t}\cos(t) - e^{2t}\cos(t)\right]$

$= \frac{1}{2}\cos(t)\left[e^{-2t} - e^{2t}\right] \Rightarrow \frac{e^{2t} - e^{-2t}}{2} = \cosh(2t)$

$x_e(t) = \cosh(2t)\cos(t)$

$x_o(t) = -\sinh(2t)\cos(t)$

Q (12) :-

ⓐ $x(t) = \cos(t) + \sin(t) + \sin(t)\cos(t)$

$\quad\quad\quad\quad e \quad\quad 0 \quad\quad 0.e=0$

$x_e(t) = \cos(t)$

$x_o(t) = \sin(t) + \sin(t)\cos(t)$

Another solution:- $x_e(t) = \frac{1}{2}[x(t) + x(-t)]$

$= \frac{1}{2}[\cos(t) + \sin(t) + \sin(t)\cos(t) + \cos(-t) + \sin(-t) + \sin(-t)\cos(-t)]$

$= \frac{1}{2}[\cos(t) + \cancel{\sin(t)} + \cancel{\sin(t)\cos(t)} + \cos(t) - \cancel{\sin(t)} - \cancel{\sin(t)\cos(t)}]$

$= \frac{1}{2}[2\cos(t) = \cos(t)$

$x_o(t) = \frac{1}{2}[x(t) - x(-t)]$

$x_o(t) = \frac{1}{2}[\cancel{\cos(t)} + \sin(t) + \sin(t)\cos(t) - \cancel{\cos(t)} + \sin(t) + \sin(t)\cos(t)]$

$x_o(t) = \frac{1}{2}[2\sin(t) + 2\sin(t)\cos(t)]$

$x_o(t) = \sin(t) + \sin(t)\cos(t)$.

ⓑ $x(t) = 1 + t + 3t^2 + 5t^3 + 9t^4$ *

$\quad\quad\quad\quad e \quad 0 \quad e \quad 0 \quad e$

* 1 is even because it is on form $1t^0$)

~~t is even~~

$x_e(t) = 1 + 3t^2 + 9t^4$

$x_o(t) = t + 5t^3$

ⓒ $x(t) = 1 + t\cos(t) + t^2\sin(t) + t^3\sin(t)\cos(t)$

$e + 0.e \quad + \quad e.0 \quad + \quad 0.0.e$

$e + 0 \quad + \quad 0 \quad + e$

$x_e(t) = 1 + t^3\sin(t)\cos(t)$

$x_o(t) = t\cos(t) + t^2\sin(t).$

ⓓ $x(t) = (1+t^3)\cos^3(10t) = \cos^3(10t) + t^3\cos^3(10t)$

$e \qquad 0.e=e$

$x_e(t) = \cos^3(10t)$

$x_o(t) = t^3\cos^3(10t)$

Q13: ⓐ $x(t) = \cos^2(2\pi t)$

$$x(t) = \frac{1+\cos(4\pi t)}{2}$$

⇒ $\frac{1}{2}$ is constant with arbitary period (can take any number) It does not affect the periodicity of the sinusoidal signal.

⇒ $\frac{\cos(4\pi t)}{2}$ is periodic with $\omega = 4\pi \Rightarrow T_o = \frac{2\pi}{4\pi} = 0.5$

So the signal $x(t) = \cos^2(2\pi t)$ is periodic with $T_o = 0.5$ (sec)

* ⓑ $\sin^2(2t) = \frac{1}{2} - \frac{\cos(4t)}{2}$

It is periodic with $\omega = 4 \Rightarrow T_o = \frac{2\pi}{4} = \frac{\pi}{2}$ sec

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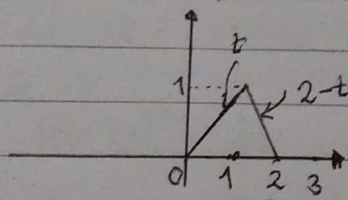
ⓒ $e^{-2t}\cos(2\pi t)$

$\cos(2\pi t)$ it self is periodic, but multiplying it with exponentially decaying number e^{-2t}, will make it decrease for all values as t increases.

So, $x(t) = e^{-2t}\cos(2\pi t)$ is non-periodic.

Q 13: ⓐ $x(t) = \begin{cases} t, & 0 \le t \le 1 \\ 2-t, & 1 \le t \le 2 \\ 0, & \text{otherwise.} \end{cases}$



The signals that have values for limited time duration and not periodic are usually Energy Signals.

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 \, dt = \int_0^1 (t)^2 \, dt + \int_1^2 (2-t)^2 \, dt$$

$$E_x = \frac{t^3}{3}\Big|_0^1 + \left(4t - \frac{4t^2}{2} + \frac{t^3}{3}\right)\Big|_1^2 = \frac{1}{3} + 4(2) - 2(2)^2 + \frac{8}{3} - 4(1) + 2(1) - \frac{1}{3}$$

$$E_x = \cancel{\tfrac{1}{3}} + \cancel{8} - \cancel{8} + \frac{8}{3} - 4 + 2 - \cancel{\tfrac{1}{3}} = \frac{8}{3} - 2 = \frac{2}{3} \text{ Joule}$$

$\frac{2}{3} < \infty$, So it is Energy signal.

* ⓑ $x(t) = 5\cos(\pi t) + \sin(5\pi t), \ -\infty < t < \infty$

The signal is a sum of two sinusoidal signals we check the periodicity.

$x_1(t) = 5\cos(\pi t) \Rightarrow \omega_1 = \pi \ , T_1 = \frac{2\pi}{\pi} = 2\,(sec)$

$x_2(t) = \sin(5\pi t) \Rightarrow \omega_2 = 5\pi \ , T_2 = \frac{2\pi}{5\pi} = \frac{2}{5}\,(sec)$

$\frac{T_1}{T_2} = \frac{2}{\frac{2}{5}} = 5 \Rightarrow$ rational number.

The signal is periodic with $T_0 = L.C.M\left(2, \frac{2}{5}\right) = 2$

$T_0 = 2(sec)$.

Thus, the signal is power signal umostly.

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} |x(t)|^2 . dt$$

$$P = \frac{1}{T_0} \int_0^{T_0} |x(t)|^2 . dt = \frac{1}{2} \int_0^2 \left(5\cos(\pi t) + \sin(5\pi t)\right)^2 . dt$$

$$P = \frac{1}{2} \int_0^2 25\cos^2(\pi t) + 10\sin(5\pi t)\cos(\pi t) + \sin^2(5\pi t) . dt$$

$$P = \frac{1}{2} \int_0^2 \left(25\left(\frac{1 + \cos(2\pi t)}{2}\right) + 10\sin(5\pi t)\cos(\pi t) + \frac{1}{2} - \frac{\cos(10\pi t)}{2}\right) . dt$$

$$P = \frac{1}{2} \int_0^2 \left(13 + \frac{\cos(2\pi t)}{2} + \underbrace{10\sin(5\pi t)\cos(\pi t)} - \frac{\cos(10\pi t)}{2}\right) . dt$$

$$\sin(5\pi t)\cos(\pi t) = \frac{\sin(5\pi + \pi)t}{2} + \frac{\sin(5\pi - \pi)t}{2}$$

$$= \frac{\sin(6\pi t)}{2} + \frac{\sin(4\pi t)}{2}$$

* Using $\Rightarrow \sin(a)\cos(b) = \frac{\sin(a+b) + \sin(a-b)}{2}$

$$P = \frac{1}{2}\int_0^2 \left(13 + \frac{\cos(2\pi t)}{2} + \frac{\sin(6\pi t)}{2} + \frac{\sin(4\pi t)}{2} - \frac{\cos(10\pi t)}{2}\right).dt$$

$$P = \frac{1}{4}\int_0^2 \left(26 + \cos(2\pi t) + \sin(6\pi t) + \sin(4\pi t) - \cos(10\pi t)\right).dt$$

$$P = \frac{1}{4}\left[26t + \frac{\sin(2\pi t)}{2\pi} - \frac{\cos(6\pi t)}{6\pi} - \frac{\cos(4\pi t)}{4\pi} - \frac{\sin(10\pi t)}{10\pi}\right]_0^2$$

$$P = \frac{1}{4}\left[26(2) + \frac{\sin(4\pi)}{2\pi} - \frac{\cos(12\pi)}{6\pi} - \frac{\cos(8\pi)}{4\pi} - \frac{\sin(20\pi)}{10\pi}\right.$$

$$\left. - \left(26(0) + \frac{\sin(0)}{2\pi} - \frac{\cos(0)}{6\pi} - \frac{\cos(0)}{4\pi} - \frac{\sin(0)}{10\pi}\right]$$

$$P = \frac{1}{4}\left[52 + 0 - \cancel{\frac{1}{6\pi}} - \cancel{\frac{1}{4\pi}} - 0 - 0 - 0 + \cancel{\frac{1}{6\pi}} + \cancel{\frac{1}{4\pi}} - 0\right]$$

$P = \frac{52}{4} = 13$ watt, $13 < \infty$, so Power signal.

(c) $x(t) = \begin{cases} 5\cos(\pi t), & -1 < t < 1 \\ 0, & \text{otherwise} \end{cases}$

The signal is bounded and has value for limited time $\Rightarrow$ Energy signal.

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 . dt = \int_{-1}^{1} (5\cos(\pi t))^2 . dt$$

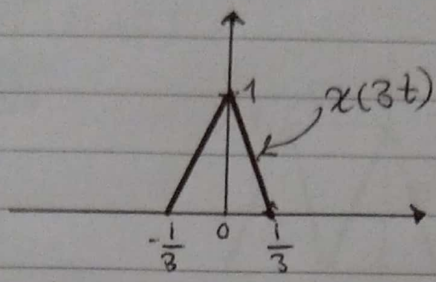
$$= 25\int_{-1}^{1} \cos^2(\pi t) . dt = \frac{25}{2}\int_{-1}^{1} (1 + \cos(2\pi t)) . dt$$

$$E = \frac{25}{2}\left[t + \frac{\sin(2\pi t)}{2\pi} \right]_{-1}^{1} = \frac{25}{2}\left[1 + \frac{\sin(2\pi)}{2\pi} + 1 + \frac{\sin(2\pi)}{2\pi} \right]$$

$$E = \frac{25}{2}(2) = 25 \text{ Joules}$$

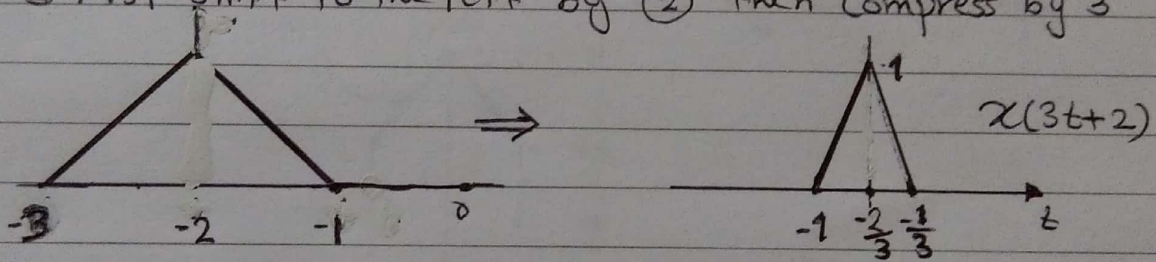
$25 < \infty$, Energy signal.

Q15: (a) $x(3t)$



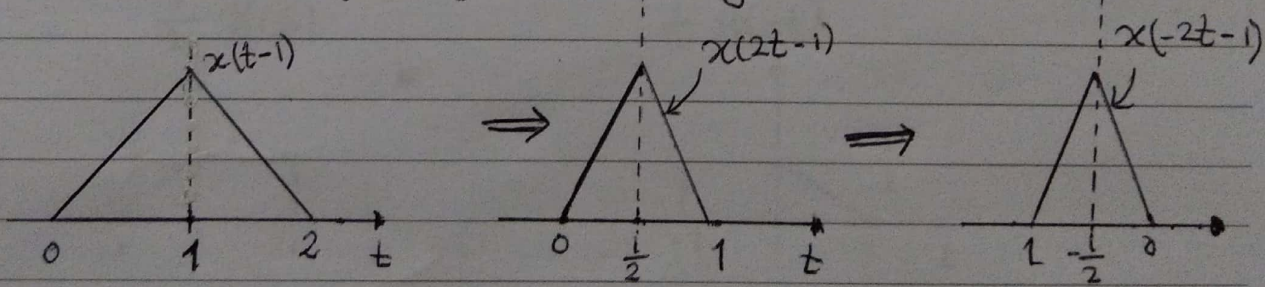
(b) $x(3t+2)$

→ First shift to the left by (2) then compress by 3

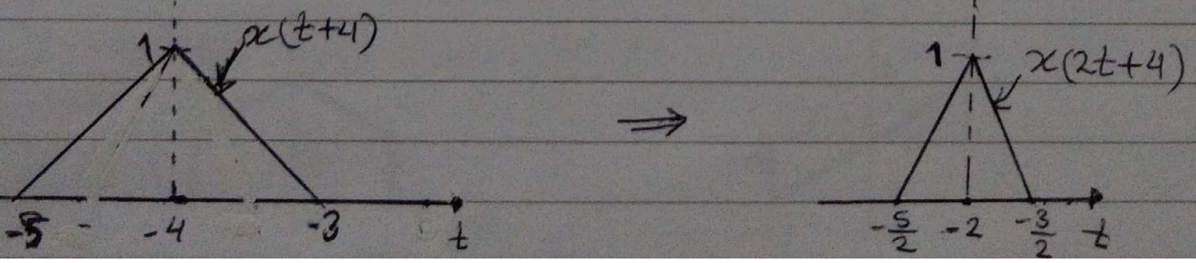


(c) $x(-2t-1)$

Shift to the right by (1) → scale by (2) and reverse.

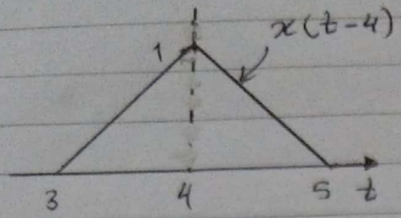


(d) $x(2(t+2)) = x(2t+4)$ shift to right by (4) and scale by (2)

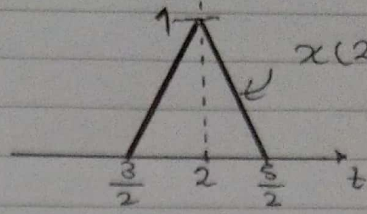


(e) $x(2(t-2)) = x(2t-4)$

shift to the right by 4 and scale by (2).

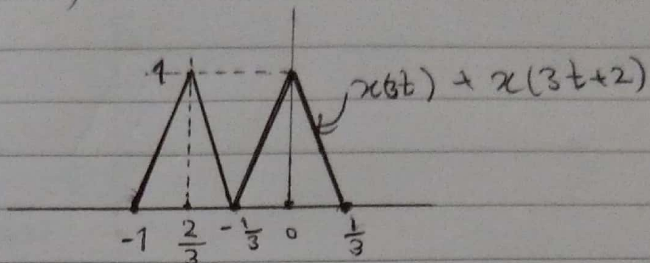


$\Rightarrow$



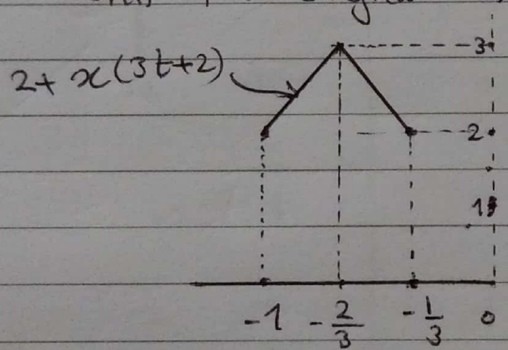
(f) $f(t) = x(3t) + x(3t+2)$

$=$ (a) $+$ (b)



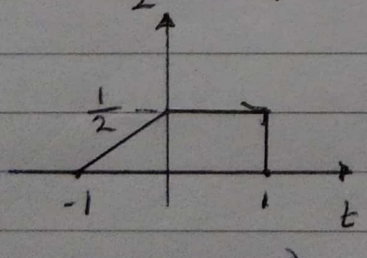
(g) $2 + x(3t+2)$

shif the signal in (b) by 2 units up

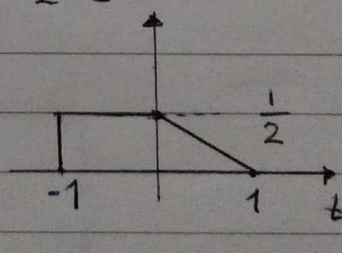


Q16: -

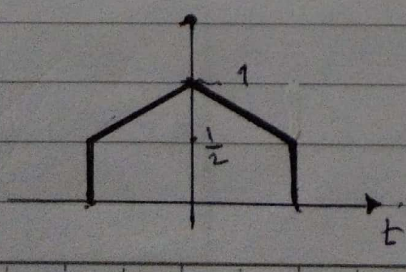
$\frac{1}{2}\, y(t)$



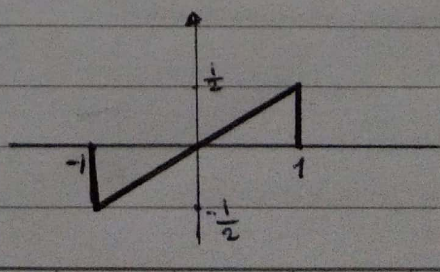
$\frac{1}{2}\, y(-t)$



$y_e(t) = \frac{1}{2}\, y(t) + \frac{1}{2}\, y(-t)$

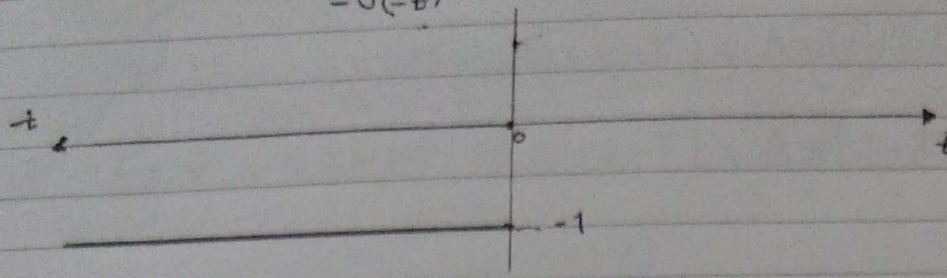


$y_o(t) = \frac{1}{2}\, y(t) - \frac{1}{2}\, y(-t)$

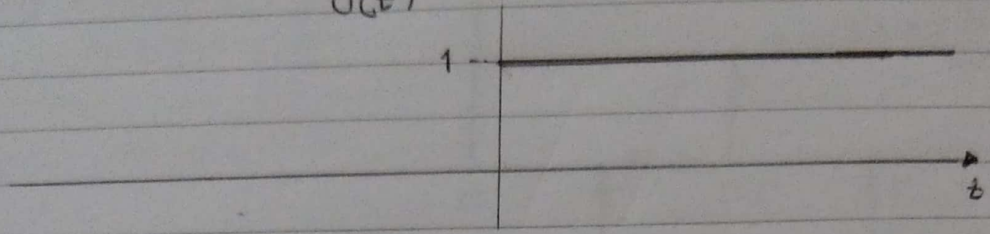


Q17 :-

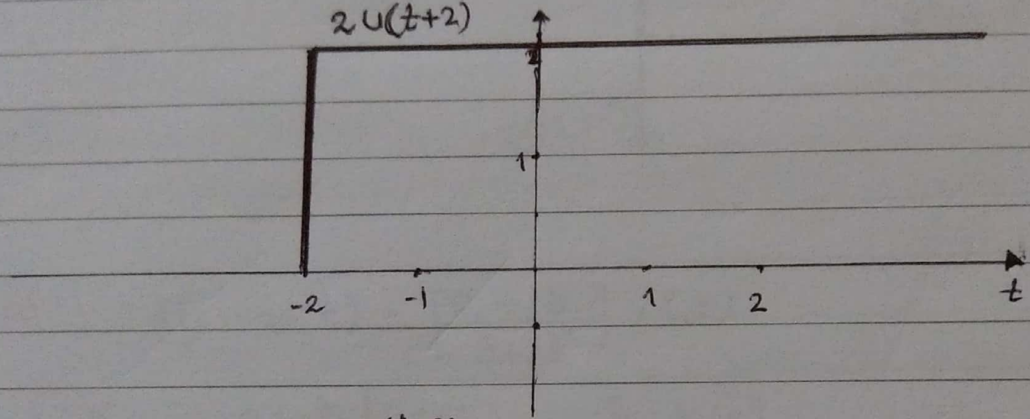
$-u(-t)$



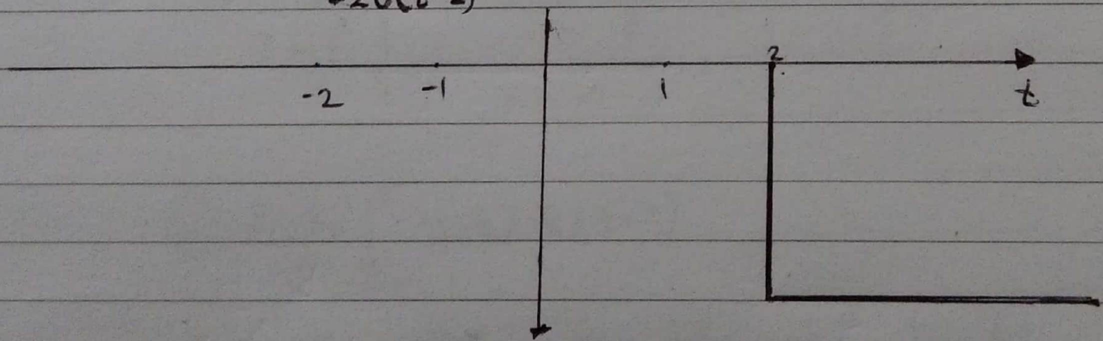
$u(t)$



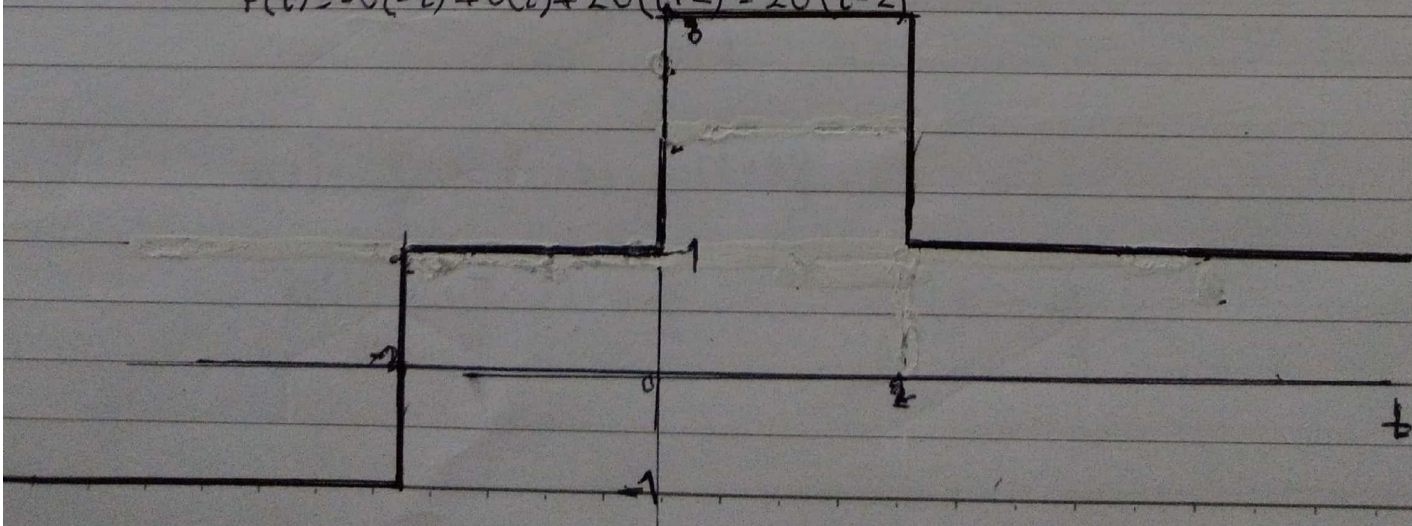
$2u(t+2)$



$-2u(t-2)$



$f(t) = -u(-t) + u(t) + 2u(t+2) - 2u(t-2)$

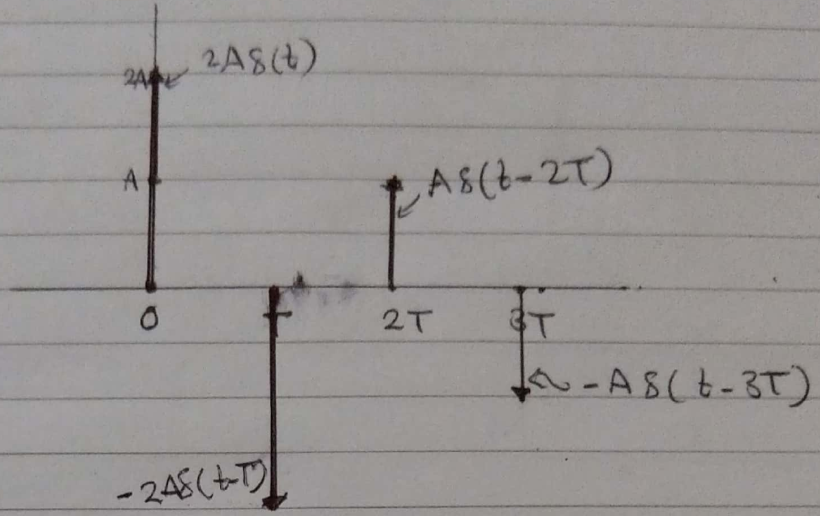


Q18:-

$$x(t) = 2A[u(t) - u(t-T)] + A[u(t-2T) - u(t-3T)]$$

$$x(t) = 2A\,u(t) - 2A\,u(t-T) + A\,u(t-2T) - A\,u(t-3T)$$

$$\frac{dx(t)}{dt} = 2A\delta(t) - 2A\delta(t-T) + A\delta(t-2T) - A\delta(t-3T)$$



Q19:- (a) $x(t) = \left[\frac{2+jt}{3+jt^2}\right] \cdot \delta(t)$

$$= \left.\frac{2+jt}{3+jt^2}\right|_{t=0} \cdot \delta(t) = \frac{2}{3}\,\delta(t)$$

(b) $f(t) = \int_{-\infty}^{\infty} y(t)\,\delta(t-2)$

$t=2$ is within the integration limits.

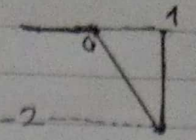
Thus $f(t) = y(t)\big|_{t=2}$

$$y(t) = \frac{6}{8}\,t\,[u(t) - u(t-8)]$$

$$y(t)\big|_{t=2} = \frac{6}{8}(2) = \frac{3}{2} \qquad f(t) = \frac{3}{2}$$

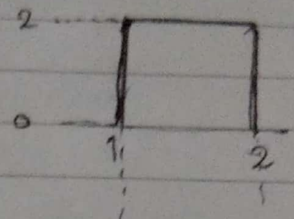
Q20:-

for first section



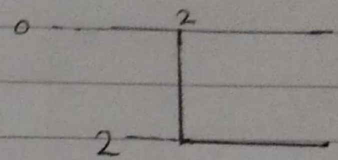
$$\frac{y_2-y_1}{x_2-x_1} = \frac{y-y_1}{x-x_1} \Rightarrow \frac{-2-0}{1-0} = \frac{y-0}{x-0}$$

$$\frac{y}{x} = -2 \Rightarrow y = -2x \Rightarrow x(t) = -2t\left[u(t) - u(t+1)\right]$$



for second section:

$$x(t) = 2\left[u(t-1) - u(t-2)\right]$$



for third section:

$$x(t) = -2u(t+2)$$

* so the signal can be expressed as:-

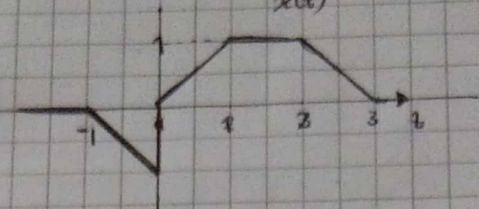
$$x(t) = -2t\,u(t) + 2t\,u(t-1) + 2u(t-1) - 2u(t-2) - 2u(t-2$$

$$x(t) = -2t\,u(t) + 2(t+1)\,u(t-1) - 4u(t-2)$$

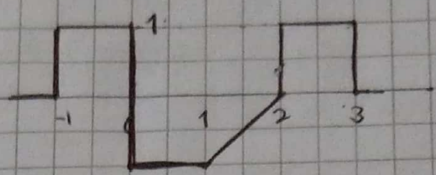
Q 21 :-

(a) $x(t)\,y(t-1)$

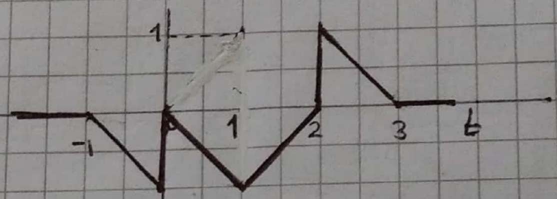
$x(t)$



$y(t-1)$

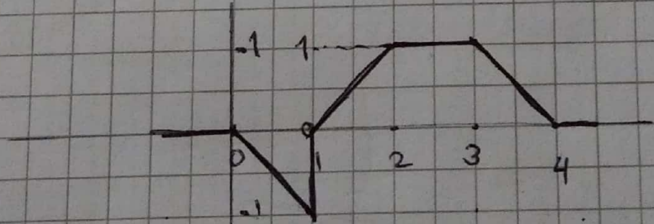


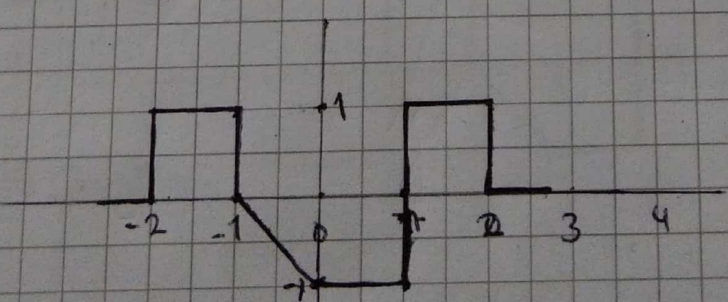
$x(t)\,y(t-1)$



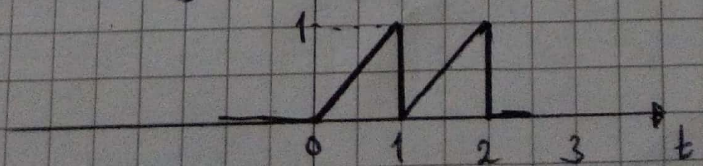
(b) $x(t-1)\,y(-t)$

$x(t-1)$

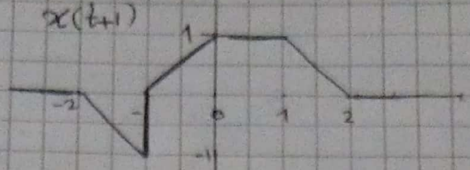


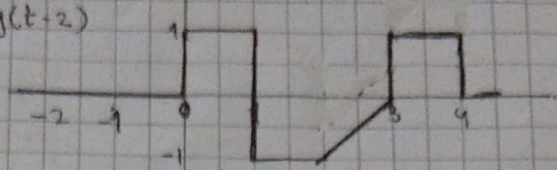


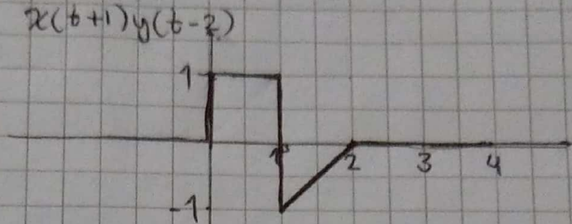
$x(t-1)\,y(-t)$

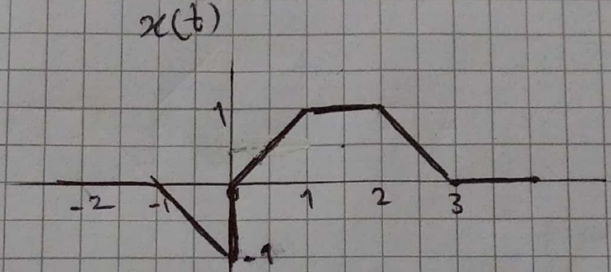


ⓒ $x(t+1)y(t-2)$

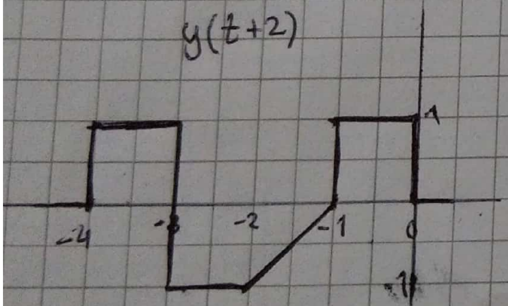


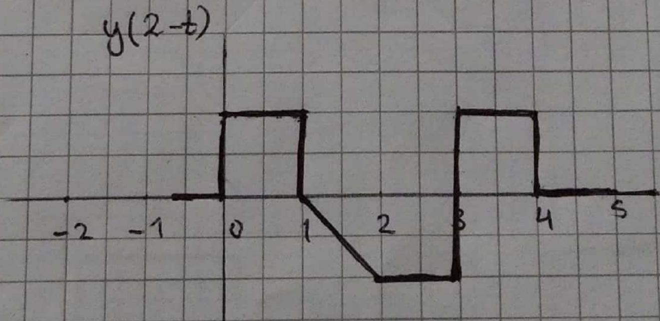


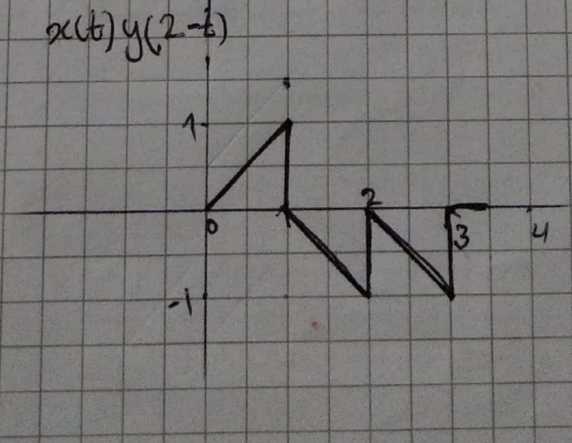




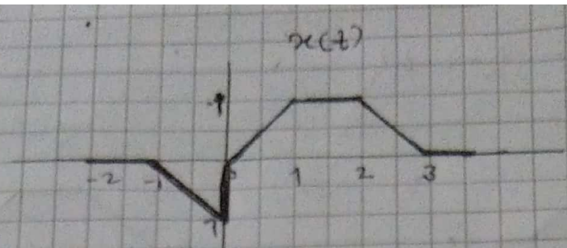
ⓓ $x(t)y(2-t)$

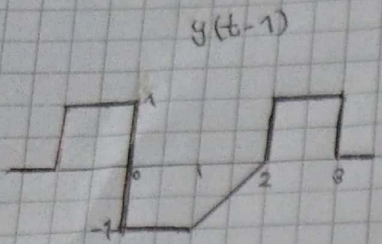


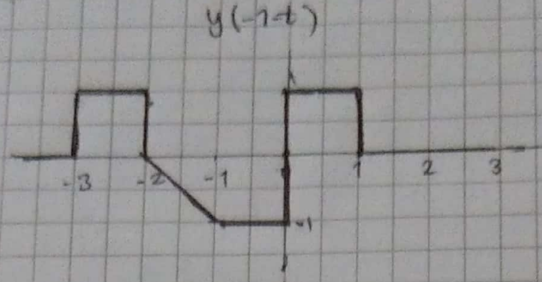


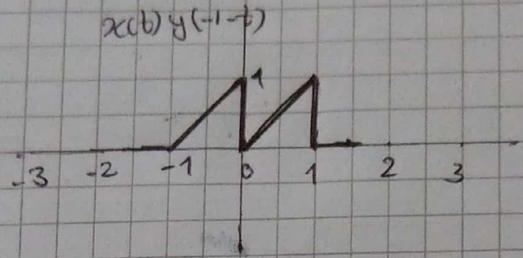


(e) $x(t)y(-1-t)$

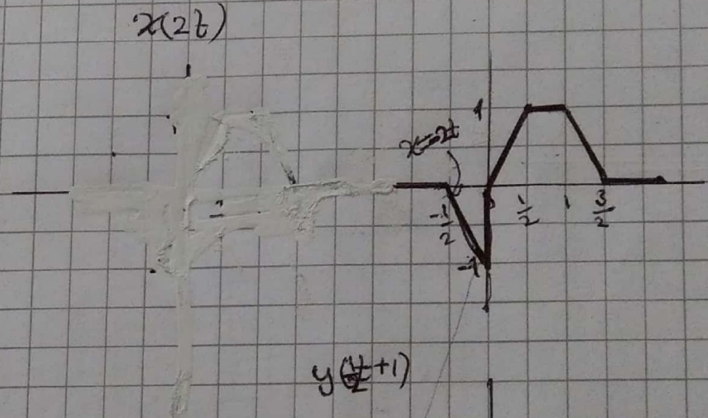


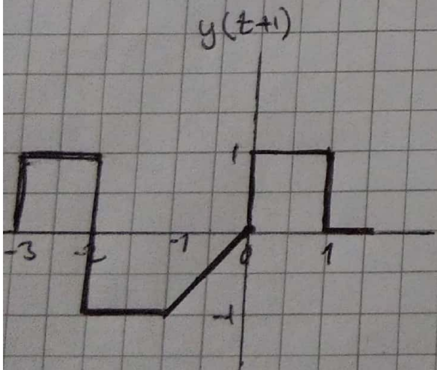


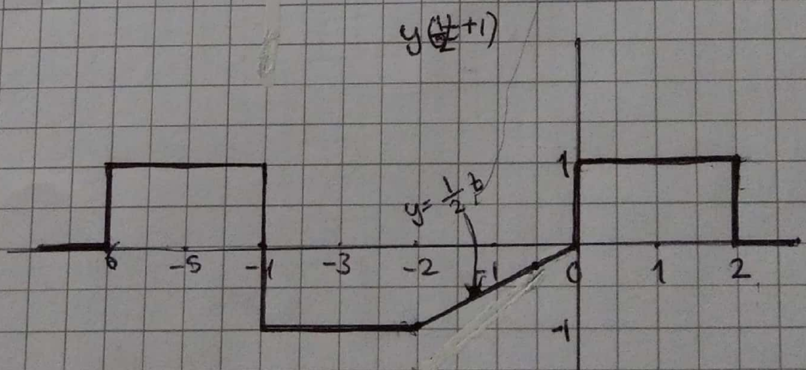


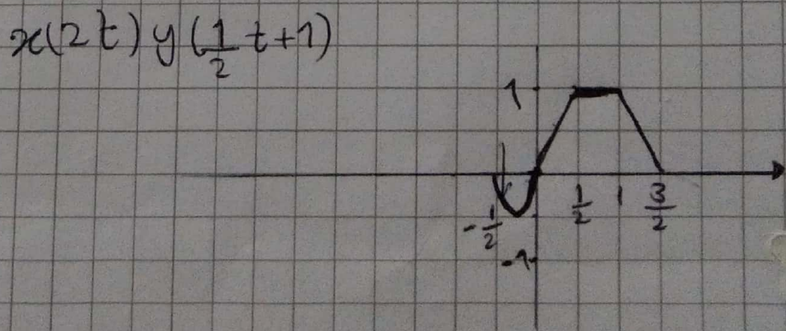


(f) $x(2t)y(\frac{1}{2}t+1)$

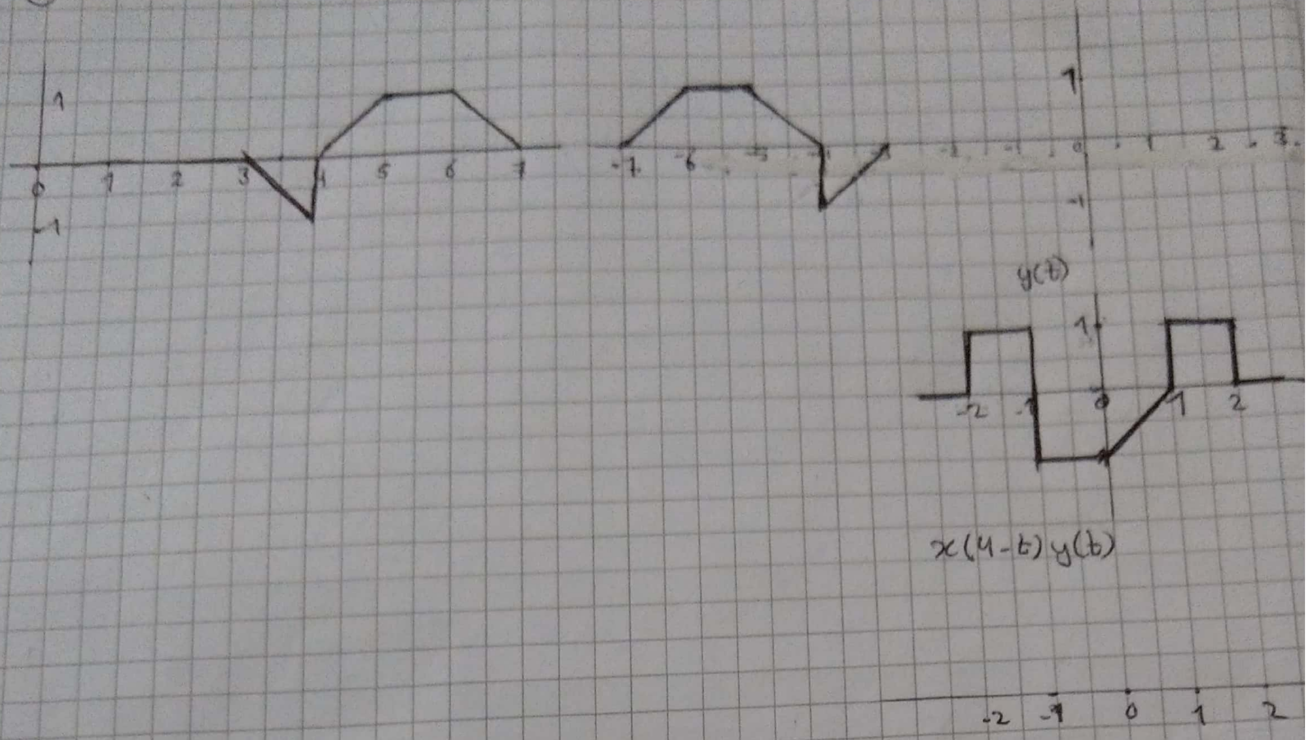




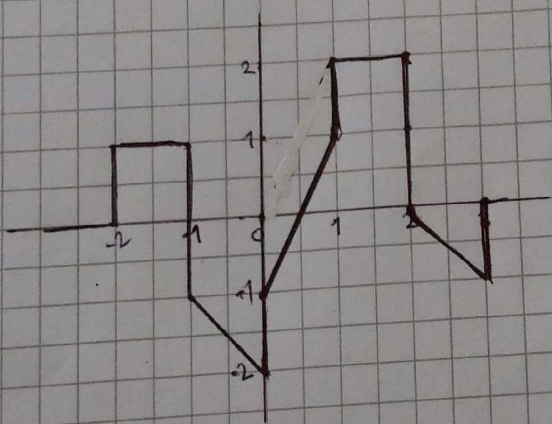




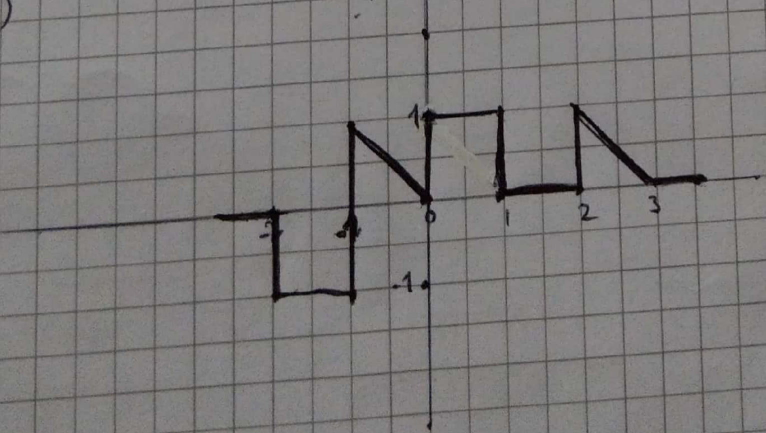
(g) $x(4-t)\,y(t)$



(h) $x(t)+y(t)$



(i) $x(t)-y(t)$



Date

Sheet #2

Q1:- * For $y(t) = x(t)\cos(\omega_0 t)$ is a "Memoryless" as the output depends only on the present value of $x(t)$.

* The output does not depend on future values of $x(t) \Rightarrow$ "Causal"

* If $|x(t)| < |M| < \infty$

Then $|\cos\omega_0 t|\,|x(t)| \leq |\cos(\omega_0 t)|\,|M| < \infty$

$y(t) < |\cos(\omega_0 t)|\cdot|M| < \infty$

$|\cos\omega_0 t| \leq 1 \Rightarrow$ Therefore the system is (BIBO stable)

* If $y_1(t) = x_1(t)\cos(\omega_c t)$, $y_2(t) = x_2(t)\cos(\omega_c t)$

Then $y_1(t) + y_2(t) = (x_1(t) + x_2(t))\cos(\omega_c t) \Rightarrow 1$

$y_3(t) = T\{x_1(t) + x_2(t)\} = (x_1(t) + x_2(t))\cos(\omega_c t) \Rightarrow 2$

$1 = 2 \Rightarrow$ The system is "linear."

* $y_{output} \Rightarrow$ shifting the signal at the output

$y_{output}(t - t_0) = x(t - t_0)\cos(\omega_0(t - t_0)) \Rightarrow 1$

$y_{input} \Rightarrow$ shifting the ~~ipt~~ input signal before operator $\Rightarrow y_{input}(t - t_0) = x(t - t_0)\cos(\omega_c t) \Rightarrow 2$

$1 \neq 2 \Rightarrow$ The system is not time invariant

Q2:- $y(t) = x(t) + 1$

The system is "non-linear" ⇒ since it is on the form $y(t) = x(t) + c$

$$y(t) = [a_1 x_1(t) + a_2 x_2(t)] + 1 \quad = a_1 x_1(t) + a_2 x_2(t) + 1$$
$$\neq (a_1 x_1(t) + 1) + (a_2 x_2(t) + 1)$$

* The system depends only on the present value of $x(t)$ so it is "memoryless"

* the system does not depend on future values of $x(t)$ ⇒ it is causal

* $y(t - t_0) = x(t - t_0) + 1$

$y_{input}(t - t_0) = x(t - t_0) + 1$

⇒ The system is time invariant.

* The system is stable ⇒ as for $|x(t)| < |M| < \infty$

$|x(t)| + 1 < |M| + 1 < \infty$

$y(t) < |M| + 1 < \infty$

The system is BIBO stable.

Q3:- $y(t) = x(2t)$

* The system depends on future values of $x(t)$

$y(1) = x(2 \times 1) \Rightarrow y(1) = x(2)$ ⇒ non-causal

* The system does not depend only on present value of $x(t)$ ⇒ The system has a memory ⇒ not memory-less

* If $|x(t)| \leq |M| < \infty$

$\Rightarrow |x(2t)| \leq |M| < \infty$

$y(t) \leq |M| < \infty$ ⇒ The system BIBO stable.

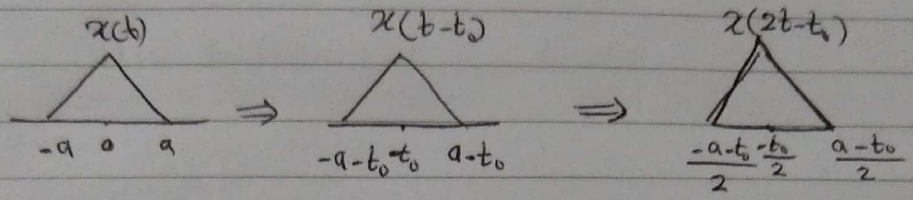
Date:

The system is not time invariant

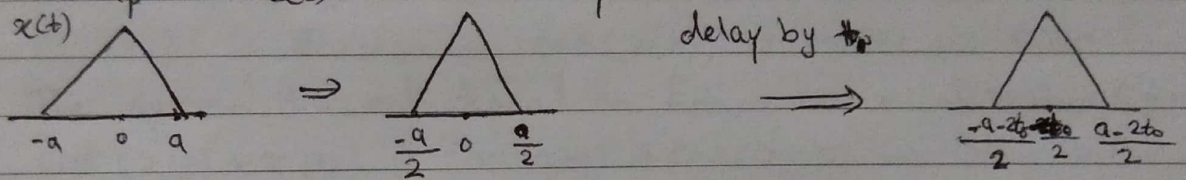
$y_{\text{output}}(t-t_0) = x(2(t-t_0)) = x(2t-2t_0)$

$y_{\text{input}}(t-t_0) = x(2t-t_0)$ $\neq$ Not Time invariant

Delay $x(t)$ by t_0 Then compress



Compress $x(t)$ then delay



$y(a_1x_1(t)+a_2x_2(t)) = a_1x_1(2t)+a_2x_2(2t) = T\{x_1(t)\}+T\{x_2(t)\}$

$= a_1x_1(2t)+a_2x_2(2t)$

The system is linear.

Q5:- if $x_1(t-t_0)$ is the input signal of the system

$y_{\text{input}}(t-t_0) = \dfrac{x_1(t-t_0)}{R(t)}$ $\neq$

* $y_{\text{output}}(t-t_0) = \dfrac{x_1(t-t_0)}{R(t-t_0)}$

The system is time variant.

Q4:- For Linearity $T\{a_1x_1(t)+a_2x_2(t)\} = a_1T\{x_1(t)\}+a_2T\{x_2(t)\}$

Date:

* $T\{a_1x_1(t)+a_2x_2(t)\} = (a_1x_1(t)+a_2x_2(t))\,(a_1x_1(t-1)+a_2x_2(t-1))$

$= a_1x_1(t)\,a_1x_1(t-1) + a_1x_1(t)\,a_2x_2(t-1) + a_2x_2(t)\,a_1x_1(t-1) + a_2x_2(t)\,a_2x_2(t-1)$

$= a_1x_1(t)\,a_1x_1(t-1) + a_2x_2(t)\,a_2x_2(t-1) + a_1x_1(t)\,a_2x_2(t-1) + a_2x_2(t)\,a_1x_1(t-1)$

$\neq$

* $T\{a_1x_1(t)\} + T\{a_2x_2(t)\} = a_1x_1(t)\,a_1x_1(t-1) + a_2x_2(t)\,a_2x_2(t-1)$

The system is nonlinear.

Q5: (a) $y(t) = \cos(x(t))$

- The system depends only on present $x(t)$ $\Rightarrow$ Memoryless.
- If $x(t)$ is stable $\Rightarrow$ $\cos(x(t))$ is stable $\Rightarrow$ BIBO stable.
- The system does not depend on future values of $x(t)$ $\Rightarrow$ Causal.
- $\cos(x_1(t)+x_2(t)) \neq \cos(x_1(t)) + \cos(x_2(t))$ $\Rightarrow$ Non linear.
- $y_{input}(t-t_0) = \cos(x(t-t_0))$, $y_{output}(t-t_0) = \cos(x(t-t_0))$ $\Rightarrow$ Time invariant.

(b) $y(t) = x(2-t)$.

- The system depends not only on current value of $x(t)$ $\Rightarrow$ Not-memoryless
- The system depends on future $x(t)$ $\Rightarrow$ not causal
- If $|x(t)| < |M| < \infty \Rightarrow |x(2-t)| < |M| < \infty$ $\Rightarrow$ BIBO stable.
- $x_1(2-t) + x_2(2-t) = T\{x_1(t)+x_2(t)\}$ $\Rightarrow$ Linear system.
- $y_{input}(t-t_0) = x(2-t-t_0)$

$\neq$

$y_{output}(t-t_0) = x(2-(t-t_0)) = x(2-t+t_0)$

The system is not Time invariant.

(c) $y(t) = x(\frac{t}{2})$

- The system depends on values other than the present value of $x(t)$ $\Rightarrow y(1) = x(\frac{1}{2}) \Rightarrow$ not memoryless
- The system ~~does n~~ depends on future values of $t \Rightarrow$ for $t<0$ $\Rightarrow y(-2) = x(-1)$ $\Rightarrow$ not causal.

Date:

- The system is stable, since the operation is just scalling and does not affect the amplitude $\Rightarrow$ BIBO stable

- For $T\{a_1 x_1(\frac{t}{2}) + a_2 x_2(\frac{t}{2})\} = a_1 x_1(\frac{t}{2}) + a_2 x_2(\frac{t}{2})$

$= a_1\{T x_1(t)\} + a_2\{T x_2(t)\}$ $\Rightarrow$ The system is linear.

- $\underset{\text{output}}{y(t-t_0)} = x\left(\frac{t-t_0}{2}\right) = x\left(\frac{t}{2} - \frac{t_0}{2}\right)$

$\neq$ The system is time invariant.

$\underset{\text{input}}{y(t-t_0)} = x\left(\frac{t}{2} - t_0\right)$

Q5:- (a) $y(t) = \frac{dx(t)}{dt}$, $x_1(t) \Rightarrow y_1(t)$, $x_2(t) \Rightarrow y_2(t)$

$a_1 x_1(t) + a_2 x_2(t) \Rightarrow \frac{d}{dt}\left[a_1 x_1(t) + a_2 x_2(t)\right] = a_1 \frac{dx_1(t)}{dt} + a_2 \frac{dx_2(t)}{dt}$

$= a_1 y_1(t) + a_2 y_2(t)$ The system is linear.

(b) $y(t) = e^{x(t)}$, $x_1(t) \rightarrow e^{x_1(t)} \Rightarrow y_1(t)$

$x_2(t) \rightarrow e^{x_2(t)} \Rightarrow y_2(t)$

$a_1 x_1(t) + a_2 x_2(t) \rightarrow e^{a_1 x_1(t) + a_2 x_2(t)} = e^{a_1 x_1(t)} \cdot e^{a_2 x_2(t)} \neq a_1 y_1(t) + a_2 y_2(t)$

The system is non linear.

(c) $y(t) = 3x(t) + 5$, $x_1(t) \rightarrow 3x_1(t) + 5 \Rightarrow y_1(t)$

$x_2(t) \rightarrow 3x_2(t) + 5 = y_2(t)$

$T\{a_1 x_1(t) + a_2 x_2(t)\} = 3a_1 x_1(t) + 3a_2 x_2(t) + 5 \neq a_1 y_1(t) + a_2 y_2(t)$

Date

Q6:- $s(t) = [1+2m(t)]\cos(2\pi\times10^8 t)$

$s_1(t) = [1+2m_1(t)]\cos(2\pi\times10^8 t)$.

$s_2(t) = [1+2m_2(t)]\cos(2\pi\times10^8 t)$

$T\{m_1(t)+m_2(t)\} \rightarrow [1+2(m_1(t)+m_2(t))]\cos(2\pi\times10^8 t)$

$s_1(t)+s_2(t) = [2+2m_2(t)]\cos(2\pi\times10^8 t) \neq T\{m_1(t)+m_2(t)\}$

The system is non-linear.

Q7:- System 1 :- $y_{input}(t-t_0) = \sin(x(t-t_0))$

$= \Rightarrow$ Time invariant

$y_{output}(t-t_0) = \sin(x(t-t_0))$

System 2 :- $y_{input}(t-t_0) = t\sin(x(t-t_0))$

$\neq$ not time Invariant

$y_{output}(t-t_0) = (t-t_0)\sin(x(t-t_0))$

Q8:- (a) $y(t) = x(t-2)$

$x_1(t) \rightarrow x_1(t-2) = y_1(t)$

$x_2(t) \rightarrow x_2(t-2) = y_2(t)$

$T\{x_1(t)+x_2(t)\} = x_1(t-2)+x_2(t-2) = y_1(t)+y_2(t)$

The system is Linear.

- $y_{input}(t-t_0) = x(t-t_0-2)$ $=$ The system is Time invariant

$y_{output}(t-t_0) = x(t-t_0-2)$

* for $|x(t)| \le M \Rightarrow |y(t)| = |x(t-2)| \le M \Rightarrow$ BIBO stable.

The shifting does not affect stability.

* The system depends on past values of $x(t) \Rightarrow$ not memoryless.

* The system depends only on past values of $x(t) \Rightarrow$ Causal.

(b) $y(t) = x(2t-5)$

$$x_1(t) \rightarrow x_1(2t-5) = y_1(t)$$

$$x_2(t) \rightarrow x_2(2t-5) \Rightarrow y_2(t)$$

$$T\{x_1(t) + x_2(t)\} = x_1(2t-5) + x_2(2t-5) = y_1(t) + y_2(t)$$

so the system is linear

→ Please note that it is on the form $y(t) = x(at \pm b)$
not $y(t) = x(t) + c$.
The shifting and scaling does not affect linearity.

* $y(t-t_0) = x(2t-5-t_0)$
input

$\neq$ The system is not Time invariant

$y(t-t_0) = x(2(t-t_0)-5) = x(2t-2t_0-5)$
output

* If $|x(t)| \le M < \infty$
Then $|y(t)| = |x(2t-5)| \le M \Rightarrow$ The system is BIBO Stable

* Shifting and Scaling does not affect stability.

* Since $y(t) = (2t-5)$ depends on values of $x(t)$ not at the current $\Rightarrow$ The system is not memoryless

- For values of $t > 5 \Rightarrow y(6) = x(2\times6-5) = x(7)$
The system will depend on future values of $x(t)$
so the system is not causal.

ⓒ $y(t) = t\,x(t+10)$

$$x_1(t) \rightarrow t\,x_1(t+10) = y_1(t)$$

$$x_2(t) \rightarrow t\,x_2(t+10) = y_2(t)$$

$$T\{x_1(t)+x_2(t)\} = t\{x_1(t)+x_2(t)\} \underset{t=t+10}{=} t\,x_1(t+10) + t\,x_2(t+10) = y_1(t)+y_2(t)$$

The system is linear

- $\underset{\text{input}}{y}(t-t_0) = t\,x(t-t_0+10)$
$\underset{\text{output}}{y}(t-t_0) = (t-t_0)\,x(t-t_0+10)$
$\neq$ The system is not time invariant

- $|x(t)| \leq M \Rightarrow |y(t)| = |t|\,|x(t+10)| \leq |M|\,|t|$

But as $t \to \infty$ $|t| \to \infty$ and the system will be unbounded, so the system is not BIBO stable.

- The system depends on future values of $x(t)$
so the system is not causal and not memoryless

ⓓ $y(t) = \begin{cases} 2, & x(t) \geq 0 \\ 0, & x(t) < 0 \end{cases}$

$$x_1(t) \rightarrow 2u(x_1(t)) = y_1(t)$$
$$x_2(t) \rightarrow 2U(x_2(t)) = y_2(t)$$

$T\{x_1(t)+x_2(t)\} = 2u(x_1(t)+x_2(t)) \neq y_1(t)+y_2(t)$. The system is not linear

* please not that operator depends on the value of $x(t)$

If $x_1(t) = 5 \Rightarrow y_1(t) = 2$

$x_2(t) = -8 \Rightarrow y_2(t) = 0$

$x_1(t) + x_2(t) = 5 - 8 = -3 \Rightarrow y = 0 \neq y_1(t) + y_2(t)$

- $y_{input}(t - t_0) = 2u(x_1(t - t_0))$

  $y_{output}(t - t_0) = 2u(x(t - t_0))$

  $=$ the system is time invariant.

- For Bounded values of $x(t) \Rightarrow$ assumption

  The output always bounded $|y(t)| \leq 2 \Rightarrow$ BIBO stable.

- The system only depends on the present value of $x(t)$.

  So it is causal and memoryless.

ⓔ $y(t) = \begin{cases} 0 & , t < 0 \\ x(t) - x(t-5) & t \geq 0 \end{cases}$

$x_1(t) \rightarrow x_1(t) - x_1(t-5)\,u(t) = y_1(t)$

$x_2(t) \rightarrow x_2(t) - x_2(t-5)\,u(t) = y_2(t)$

$T\{x_1(t) + x_2(t)\} = x_1(t) - x_1(t-5) + x_2(t) - x_2(t-5)u(t) = y_1(t) + y_2(t)$

The system is linear.

$y_{input}(t - t_0) = x(t - t_0) - x(t - t_0 - 5)\,u(t)$

$y_{output}(t - t_0) = x(t - t_0) - x(t - t_0 - 5)\,u(t - t_0)$

$\neq$ Invariant

- If $|x(t)| \leq M < \infty \Rightarrow |y(t)| = |x(t) - x(t-5)| \leq |x(t)| + |x(t-5)| \leq 2M$

  The system is BIBO stable.

- The system depends on past values of $x(t) \Rightarrow$ not memory less

Date:

- The system does not depend on future values of $x(t)$ so The system is Causal.

(f) $y(t) = 7x^2(t) + 5x(t) + 3$

The system equation is second degree $\Rightarrow$ The system is non linear

$x_1(t) \rightarrow 7x_1^2(t) + 5x_1(t) + 3 = y_1(t)$

$x_2(t) \rightarrow 7x_2^2(t) + 5x_2(t) + 3 = y_2(t)$

$T\{x_1(t) + x_2(t)\} = 7(x_1(t) + x_2(t))^2 + 5(x_1(t) + x_2(t)) + 3$

$= 7x_1^2(t) + 7x_2^2(t) + 14x_1(t)x_2(t) + 5x_1(t) + 5x_2(t) + 3 \neq y_1(t) + y_2(t)$

The system is not linear.

- $y_{input}(t-t_0) = 7x^2(t-t_0) + 5x(t-t_0) + 3$

$y_{output}(t-t_0) = 7x^2(t-t_0) + 5(x(t-t_0)) + 3$

= Time invariant

- If $|x(t)| \leq M \Rightarrow |y(t)| \leq 7|x(t)||x(t)| + 5|x(t)| + 3 \leq 7M^2 + 5M + 3$

The system is BIBO stable

- The system depends only on the current value of $x(t)$ so it is Memoryless and Causal

(g) $y(t) = \mathrm{Sgn}(x(t))$

$x_1(t) \rightarrow \mathrm{sgn}(x_1(t)) = y_1(t)$

$x_2(t) \rightarrow \mathrm{sgn}(x_2(t)) = y_2(t)$

$T\{x_1(t)+x_2(t)\} = sgn\{x_1(t)+x_2(t)) \neq y_1(t)+y_2(t)$

From The defintion of sgn $y(t)\begin{cases} 1, & x(t)\geqslant 0 \\ 0, & x(t)<0 \end{cases}$

so if $x_1(t) = -4 \Rightarrow y_1(t) = 0$

$x_2(t) = 3 \Rightarrow y_2(t) = 1$

$x_1(t)+x_2(t) = -1 \Rightarrow y(t) = 0 \neq y_1(t)+y_2(t)$

The system is non-linear.

- $y_{input}(t-t_0) = sgn(x(t-t_0))$

$y_{output}(t-t_0) = sgn(x(t-t_0))$

$=$ The system is time invariant

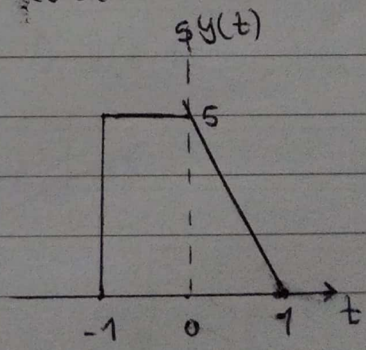
- Since The values of $y(t)$ will be either 0 or 1 or -1 and assuming Bounded input

The system is BIBO stable.

- The system depends only on the present value of $x(t)$ so it is memory-less and causal.

Q9: -

(a) $5x(t) \Rightarrow 5y(t)$



(b) The system is time invariant

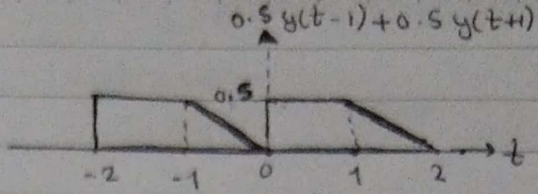
$x(t-1) \rightarrow y(t-1)$

$x(t+1) \rightarrow y(t+1)$

and because the system is linear $\Rightarrow 0.5(x(t-1))+0.5x(t+1)$

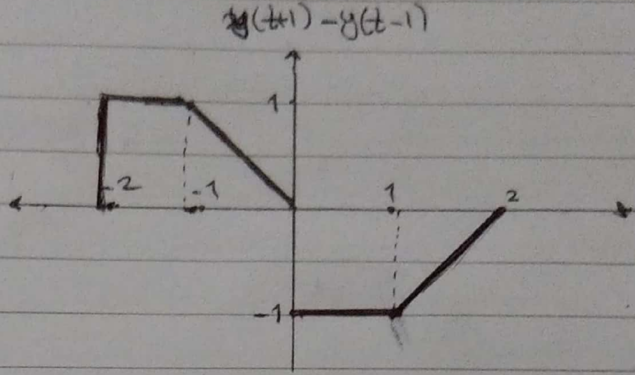
$\rightarrow 0.5y(t-1)+0.5y(t+1)$

(b)



(c) $x(t+1) - x(t-1)$

$x(t+1) \rightarrow y(t+1)$, $x(t-1) \rightarrow y(t-1)$



(d) $\frac{d\,x(t)}{dt} + 3x(t)$

$$\frac{d x(t)}{dt} \rightarrow \frac{d y(t)}{dt}$$

$$\frac{d\,x(t)}{dt} + 3x(t) \rightarrow \frac{d y(t)}{dt} + 3y(t)$$

$$x(t) = u(t+1) - u(t) + (-t+1)\left[u(t) - u(t-1)\right]$$

$$x(t) = u(t+1) - \cancel{u(t)} + \cancel{u(t)} - t\,u(t) + t\,u(t-1) - u(t-1)$$

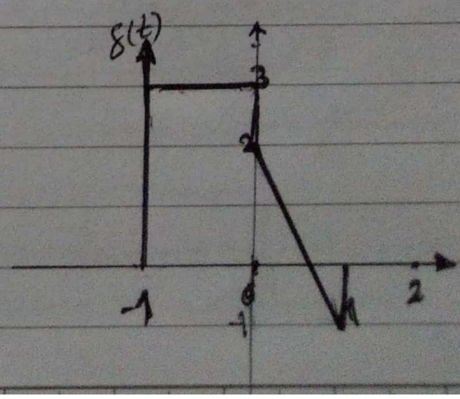
$$\frac{d\,x(t)}{dt} = \delta(t+1) - t\,\delta(t) - u(t) + t\,\delta(t-1) + u(t-1) - \delta(t-1)$$

$$\frac{d\,x(t)}{dt} = \delta(t+1) - t|_{t=0}\,\delta(t) - u(t) + t|_{t=1}\,\delta(t-1) + u(t-1) - \delta(t-1)$$

$$= \delta(t+1) - 0 - u(t) + \cancel{\delta(t-1)} + u(t-1) - \cancel{\delta(t-1)}$$

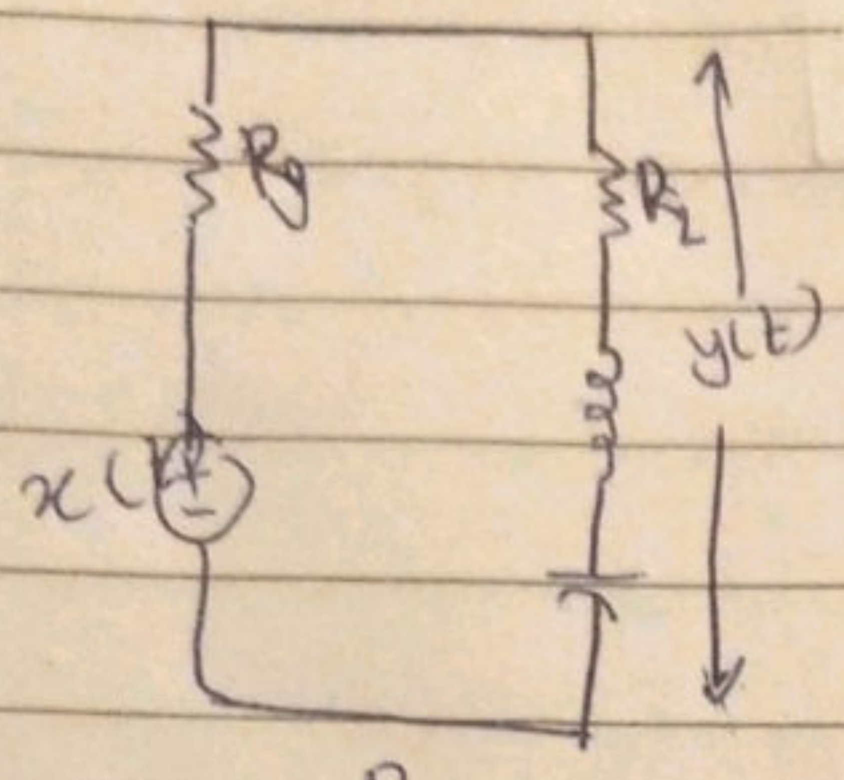
$$= \delta(t+1) - u(t) + u(t-1)$$

$$y(t) = \delta(t+1) - u(t) + u(t-1) + 3y(t)$$



Sheet 3 Q1

For the shown circuity, Find the input-output relationship as differential equation.



$$x(t) = V_{Rg} + y(t)$$

$$x(t) = iR_g + y(t)$$

$$i = \frac{x(t) - y(t)}{R_g}$$

$$V_{RL} = iR_L = \frac{R_L}{R_g}\,(x(t) - y(t))$$

$$V_L = L\frac{di}{dt} = \frac{L}{R_g}\frac{d}{dt}(x(t) - y(t)) = \frac{L}{R_g}\left[\frac{dx(t)}{dt} - \frac{dy(t)}{dt}\right]$$

$$V_c = \frac{1}{c}\int_{-\infty}^{t} i\,.dt = \frac{1}{CR_g}\int_{-\infty}^{t}(x(t) - y(t))\,.dt$$

$$x(t) = V_{R_L} + V_L + V_c + V_{Rg}$$

$$\cancel{x(t)} = \cancel{x(t)} - y(t) + \frac{R_L}{R_g}[x(t) - y(t)] + \frac{L}{R_g}\left[\frac{dx(t)}{d(t)} - \frac{dy(t)}{d(t)}\right] + \frac{1}{CR_g}\int_{-\infty}^{t} x(t) - y(t)$$

$$y(t) = \frac{R_L}{R_g}[x(t) - y(t)] + \frac{L}{R_g}\left[\frac{dx(t)}{d(t)} - \frac{dy(t)}{d(t)}\right] + \frac{1}{CR_g}\int_{-\infty}^{t}(x(t) - y(t)).\,dt$$

$$R_L = R_g = R$$

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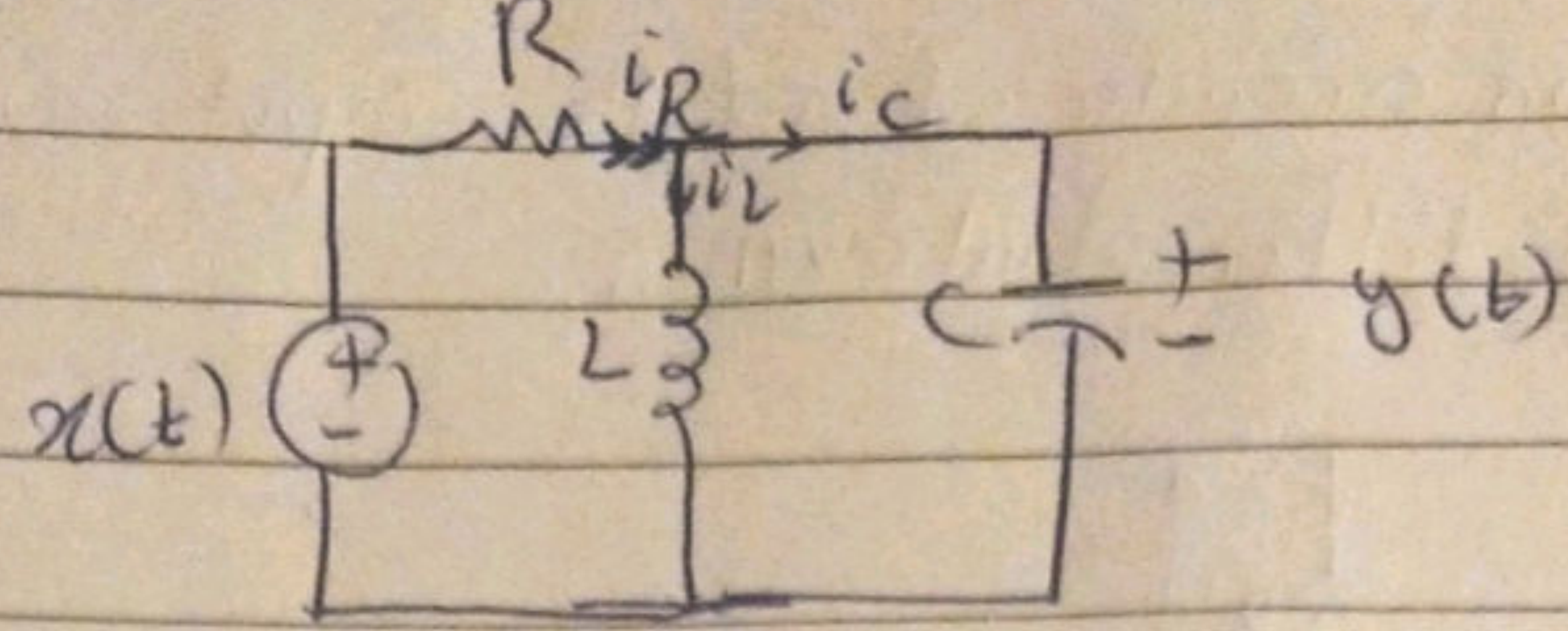
$$\frac{dy(t)}{dt} = \frac{dx(t)}{d(t)} - \frac{dy(t)}{d(t)} + \frac{L}{R}\left[\frac{d^2x(t)}{d(t)^2} - \frac{d^2y(t)}{dt^2}\right] + \frac{1}{CR}\left[x(t) - y(t)\right]$$

$$\frac{L}{R}\frac{d^2y(t)}{d(t)^2} + \frac{2\,dy(t)}{d(t)} + \frac{1}{CR}y(t) = \frac{L}{R}\frac{d^2x(t)}{d(t)^2} + \frac{dx(t)}{d(t)} + \frac{1}{CR}x(t)$$

B



$$V_c = y(t) = V_L = L\frac{di_L}{dt}$$

$$x(t) = V_R + y(t) \Rightarrow (1)$$

$$V_R = Ri_R$$

$$i_R = i_L + i_c \Rightarrow$$

$$i_c = C\frac{d(v)}{dt} = \frac{C\,dy}{d(t)}$$

$$i_L = \frac{1}{L}\int_{-\infty}^{t} V_L \, dt = \frac{1}{L}\int_{-\infty}^{t} y(t)\, dt$$

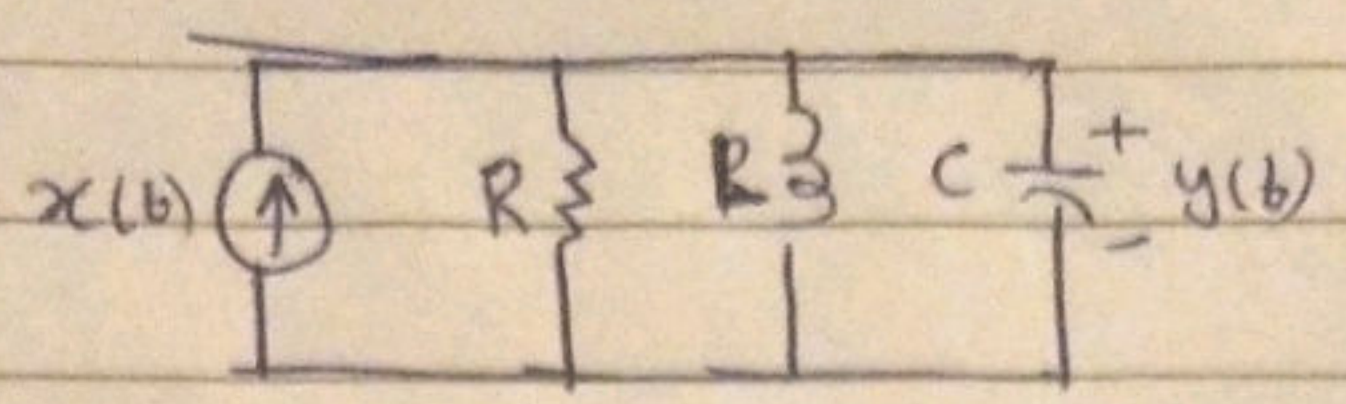
$$V_R = Ri_R = R[i_L + i_c] = RC\frac{dy(t)}{d(t)} + \frac{R}{L}\int_{-\infty}^{t} y(t)\, dt$$

$$x(t) = V_R + y(t)$$

$$x(t) = RC\frac{dy(t)}{d(t)} + \frac{R}{L}\int_{-\infty}^{t} y(t)\, dt + y(t)$$

$$\frac{dx(t)}{d(t)} = RC\frac{d^2y(t)}{dt^2} + \frac{R}{L}y(t) + \frac{dy(t)}{d(t)}$$ ✓

③



$$x(t) = i_R + i_R + i_C$$

$$V_R = V_L = V_C = y(t)$$

$$i_C = C\frac{dv_C(t)}{d(t)} = C\frac{dy(t)}{d(t)}$$

$$V_R = R i_R = y(t) \Rightarrow i_R = \frac{y(t)}{R}$$

$$i_L = \frac{1}{L}\int_{-\infty}^{t} V_L \, . \, dt = \frac{1}{L}\int_{-\infty}^{t} y(t) \, . \, dt$$

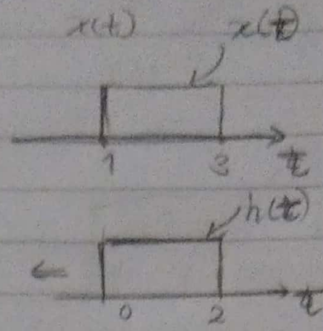
$$x(t) = i_R + i_L + i_C$$

$$x(t) = \frac{y(t)}{R} + \frac{1}{L}\int_{-\infty}^{t} y(t) \, . \, dt + C\frac{dy(t)}{d(t)}$$

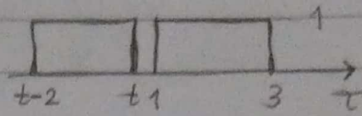
$$\frac{dx(t)}{dt} = \frac{1}{R}\frac{dy(t)}{d(t)} + \frac{1}{L} y(t) + C\frac{d^2y(t)}{d(t)^2}$$

Q2:- (a) $x(t) = u(t-1) - u(t-3)$ $\quad h(t) = u(t) - u(t-2)$

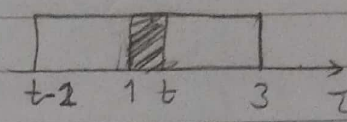
$y(t) \in (1, 5)$



- for $t<1 \Rightarrow$ no overlapping
  $y(t) = 0$

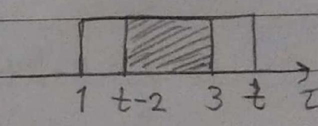


- For $t>1$, $t-2<1$
  $1<t<3$



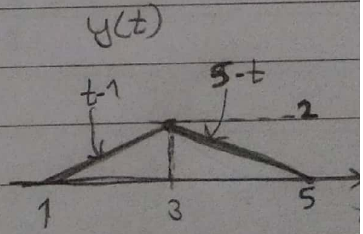
$$y(t) = \int_1^t 1.1 \, d\tau = \tau \Big|_1^t = t-1$$

for $t>3$, $t-2<3$
$3<t<5$



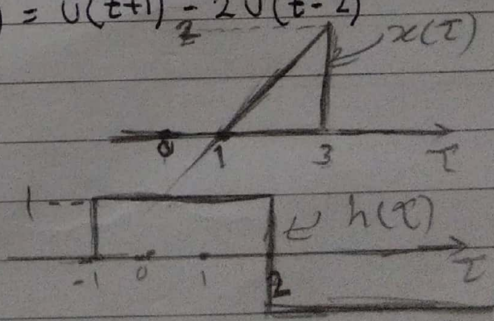
$$y(t) = \int_{t-2}^{3} 1.1 \, d\tau = \tau \Big|_{t-2}^{3} = 3-t+2 = 5-t$$

$$y(t) = \begin{cases} 0, & t<1 \\ t-1, & 1<t<3 \\ 5-t; & 3<t<5 \\ 0 & ,5<t \end{cases}$$



(b) $x(t) = (t-1)[u(t-1) - u(t-3)]$, $h(t) = u(t+1) - 2u(t-2)$

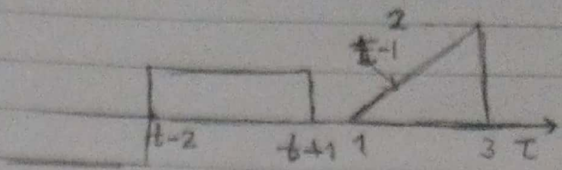
$y(t) \in (0, \infty)$



for $t+1<1$ $t<0$

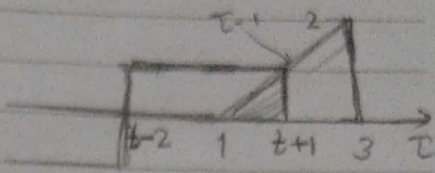
No over lapping

$y(t)=0$



For $t+1>1$, $t+1<3$

$0<t<2$

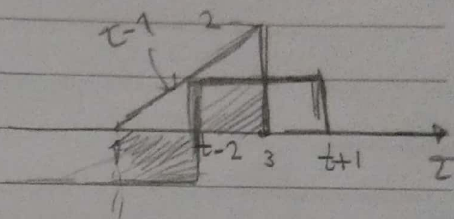


$$y(t)=\int_{1}^{t+1}(\tau-1)\,d\tau=\left.\frac{\tau^2}{2}-\tau\right|_{1}^{t+1}=\frac{(t+1)^2}{2}-t-\cancel{1}-\frac{1}{2}+\cancel{1}$$

$$y(t)=\frac{t^2+\cancel{2t}+\cancel{1}-\cancel{2t}-\cancel{1}}{2}=\frac{t^2}{2}$$

for $t+1>3$, $t-2<3$

$2<t<5$



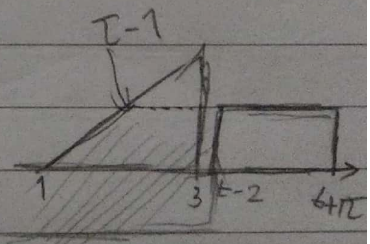
$$y(t)=\int_{1}^{t-2}-1(\tau-1)+\int_{t-2}^{3}(\tau-1)\,d\tau$$

$$y(t)=\left.\frac{-\tau^2}{2}+\tau\right|_{1}^{t-2}+\left.\frac{\tau^2}{2}-\tau\right|_{t-2}^{3}=\frac{-(t^2-4t+4)}{2}+t-2+\frac{1}{2}-1+\frac{9}{2}-3-\frac{(t^2-4t+4)}{2}+t-2$$

$$y(t)=\frac{-t^2+4t-4-t^2+4t-4}{2}+t-2+t-2+\frac{1}{2}-1+\frac{9}{2}-3$$

$$y(t)=-t^2+4t-4+2t-4+5-4=-t^2+6t+7$$

For $t-2>3 \Rightarrow t>5$, $5<t<\infty$

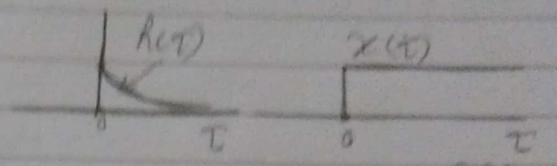


$$y(t)=\int_{1}^{3}-(\tau-1)\,d\tau=\int_{1}^{3}(-\tau+1)\,d\tau$$

$$y(t)=\left.\frac{-\tau^2}{2}+\tau\right|_{1}^{3}=\frac{-9}{2}+3+\frac{1}{2}-1=-2$$

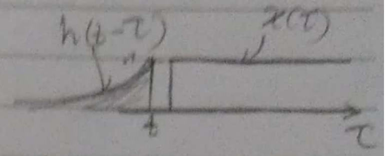
$$y(t) = \begin{cases} 0, & t \le 0 \\ t^2, & 0<t<[illegible] \\ -t^2+[illegible]-1, & [illegible]<t<[illegible] \\ =[illegible], & t>[illegible] \end{cases}$$

(c) $x(t) = u(t)$, $h(t) = e^{-t} u(t)$



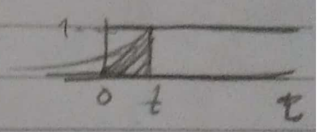
For $t<0 \Rightarrow$ no overlapping

$y(t) = 0$

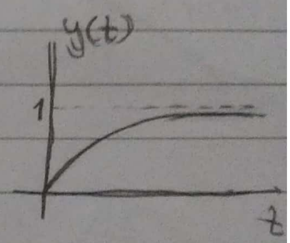


For $t>0$

$$y(t) = \int_0^t e^{-t} . e^{\tau} . d\tau$$



$$y(t) = e^{-t} \left| \frac{e^{\tau}}{1} \right|_0^t = e^{-t}\left[e^{t} - e^{0}\right] = \left[1 - e^{-t}\right] u(t)$$



Q3:- (a) The system is not memoryless as $h(t)$ not in the form $h(t) = c\delta(t)$

. $h(t)$ is not zero for $t<0 \Rightarrow$ not causal.

$$S = \int_{-1}^{1} |h(t)| . dt = \int_{-1}^{1} 1 . dt = t\Big|_{-1}^{1} = 1+1 = 2.$$

So the system is BIBO stable.

(b) The system is not memoryless.

. $h(t)$ is zero for $t<0 \Rightarrow$ causal

$$S = \int_0^1 1 . dt + \int_1^{\infty} -1 . dt = t\Big|_0^1 - t\Big|_1^{\infty} = 1+1-\infty \Rightarrow \infty$$

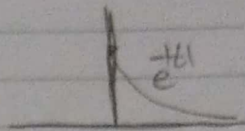
as $t \to \infty \Rightarrow h(t) \to \infty$

The system is not BIBO stable.

(c) The system is not memoryless.

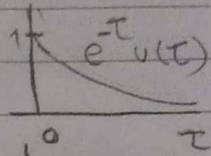
- The system is causal because $h(t)$ is zero for $t<0$

- $S = \int_0^{\infty} e^{-t}\,dt = -e^{-t}\Big|_0^{\infty} = e^{0} - e^{-\infty} = 1$



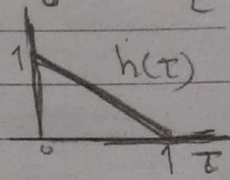
The system is BIBO stable.

Q4:- $x(t) = e^{-t}u(t)$, $h(t) = \begin{cases} 1-t, & 0 \le t \le 1 \\ 0, & \text{otherwise} \end{cases}$



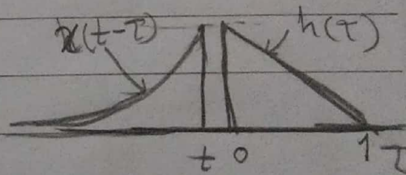
$y(t) \in (0, \infty)$

$x(t-\tau) = e^{-(t-\tau)}u(t-\tau) = e^{-t} \cdot e^{\tau} u(t-\tau)$

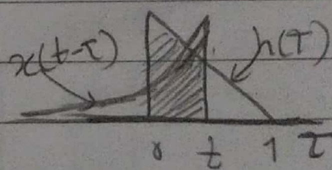


For $t<0 \Rightarrow$ no overlapping

$y(t) = 0$



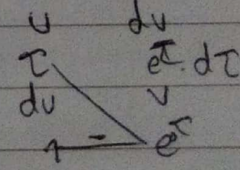
For $t>0$, $t<1$, $0<t<1$



$y(t) = \int_0^t e^{-t} \cdot e^{\tau} \cdot (1-\tau) \cdot d\tau$

$y(t) = e^{-t} \int_0^t (e^{\tau} - \tau e^{\tau}) \cdot d\tau = e^{-t}\, e^{\tau}\Big|_0^t - e^{-t}\int_0^t \tau e^{\tau} \cdot d\tau$

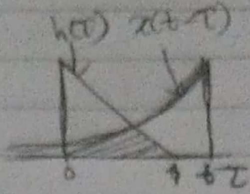
$\int_0^t \tau e^{\tau} \cdot d\tau = \tau e^{\tau} - \int_0^t e^{\tau} \cdot d\tau = \tau e^{\tau} - e^{\tau}\Big|_0^t$



$y(t) = e^{-t}\left[e^{\tau} - \tau e^{\tau} + e^{\tau}\right]_0^t = e^{-t}\left[2e^{\tau} - \tau e^{\tau}\right]_0^t = e^{-t}\left[2e^{t} - te^{t} - 2\right]$

$$y(t) = e^{-t}\left[2e^{t} - te^{t} - 2\right] = 2 - t - 2e^{-t}$$

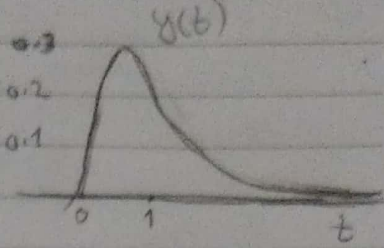
For $t \geq 1$



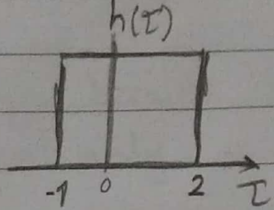
$$y(t) = \int_0^1 e^{-t}.e^{\tau}(1-\tau).d\tau = e^{-t}\left[2e^{\tau} - \tau e^{\tau}\right]_0^1$$

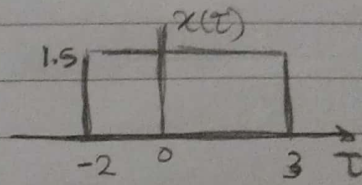
$$y(t) = e^{-t}\left[2e - e - 2\right] = e^{-t}\left[e-2\right] = e^{1-t} - 2e^{-t}$$

$$y(t) = \begin{cases} 0 & , t<0 \\ 2-t-2e^{-t} , & 0 \leq t \leq 1 \\ e^{1-t} - 2e^{-t} , & t>1 \end{cases}$$

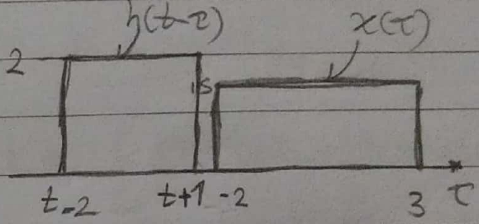


Q5:- $x(t) = \begin{cases} 1.5 & , -2 \leq t \leq 3 \\ 0 & , \text{otherwise} \end{cases}$, $h(t) = \begin{cases} 2 & , -1 \leq t \leq 2 \\ 0 & , \text{otherwise} \end{cases}$



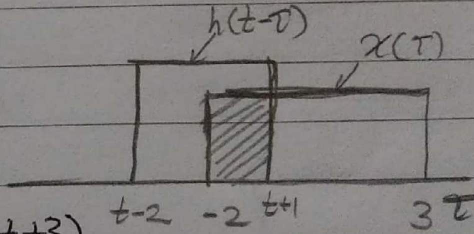


$y(t) \in (-3, 5)$



For $t+1 < -2 \Rightarrow t < -3$

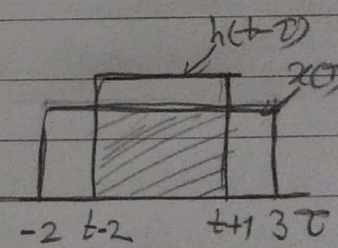
No overlapping $\Rightarrow y(t) = 0$



For $t+1 \geq -2$, $\cancel{t-2 < -2}$

$-3 \leq t \leq 0$

$$y(t) = \int_{-2}^{t+1} (1.5)(2).d\tau = 3\tau\Big|_{-2}^{t+1} = 3t+3+6 = 3(t+3)$$



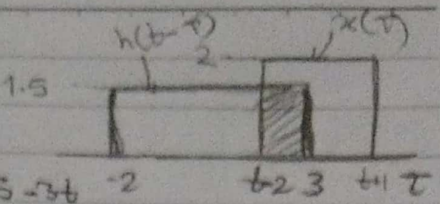
For $t+1 \leq 3$, $t-2 \geq -2$

$0 \leq t \leq 2$

$$y(t) = \int_{t-2}^{t+1} 3.d\tau = 3\tau\Big|_{t-2}^{t+1} = \cancel{3t}+3 \cancel{-3t}+6 = 9$$

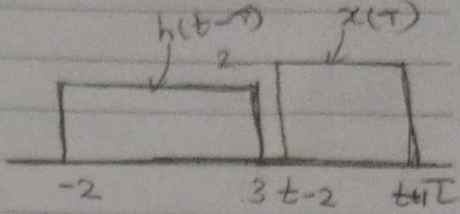
For $t+1 \geq 3$, $t-2 \leq 3$

$2 \leq t \leq 5$



$$y(t) = \int_{t-2}^{3} 3 \, d\tau = 3\tau \Big|_{t-2}^{3} = 9 - 3t + 6 = 15 - 3t$$

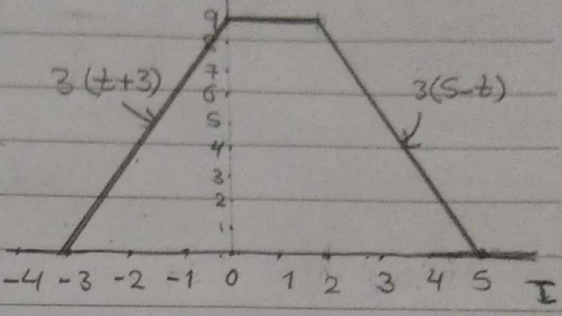
$$y(t) = 3(5-t)$$



For ~~$t>5$~~ $t-2 > 3$, $t>5$

No over-lapping , $y(t)=0$

$$y(t) = \begin{cases} 0, & t < -3 \\ 3(t+3), & -3 \leq t \leq 0 \\ 9, & 0 \leq t \leq 2 \\ 3(5-t), & 2 \leq t \leq 5 \\ 0, & t > 5 \end{cases}$$



Q6- I. $h(t) = \delta(t) - \delta(t-2)$

- The system has a memory because it depends on input not at the present only.

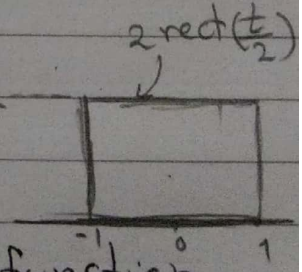
- As $h(t) = 0$ for $t<0 \Rightarrow$ system is causal.

- $$S = \int_{-\infty}^{\infty} |h(t)| \, dt = \int_{-\infty}^{\infty} |\delta(t) - \delta(t-2)| \, dt \leq \int_{-\infty}^{\infty} |\delta(t)| \, dt + \int_{-\infty}^{\infty} |\delta(t-2)| \, dt$$

$$= 1 + 1 = 2$$

The system is BIBO stable

II. $h(t) = 2\,\text{rect}\left(\frac{t}{2}\right)$



- The system has a memory as $h(t)$ is function of $\left(\frac{t}{2}\right)$. $\Rightarrow$ not memoryless

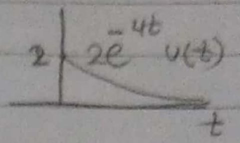
The system $\Rightarrow h(t) \neq 0$ for $t<0 \Rightarrow$ system is not causal

$$\Rightarrow \int_{-\infty}^{\infty} |h(t)|\,dt = \int_{-1}^{1} 2\,dt = 2t\Big|_{-1}^{1} = 4 < \infty$$

The system is stable.

III $h(t) = 2e^{-4t}\, u(t)$

Since $h(t) \neq 0$ for $t \neq 0$, the system is not memoryless.

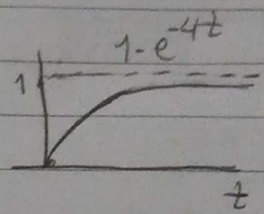


- $h(t) = 0$ for $t < 0$, system is causal,

- $$\int_{-\infty}^{\infty} |h(t)|\,dt = \int_{0}^{\infty} 2e^{-4t}\,dt = \frac{-e^{-4t}}{2}\Big|_{0}^{\infty} = \frac{e^{0}}{2} - \frac{e^{-\infty}}{2} = \frac{1}{2} < \infty$$

The system is BIBO stable.

IV :- $h(t) = (1 - e^{-4t})\, u(t)$



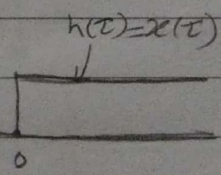
- Since $h(t) \neq 0$ for $t \neq 0 \Rightarrow$ The system is not memoryless.

- $h(t) = 0$, for $t < 0$, The system is causal.

- $$\int_{-\infty}^{\infty} |h(t)|\,dt = \int_{0}^{\infty} (1 - e^{-4t})\,dt = \left[t + \frac{e^{-4t}}{4}\right]_{0}^{\infty} = \infty$$

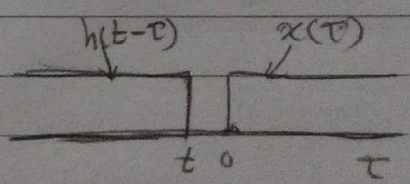
The system is not BIBO stable.

Q7:- $x(t) = u(t)$, $h(t) = u(t)$.



Since both Functions from $0 \to \infty$

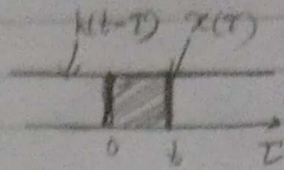
$y(t) = x(t) * h(t) \in (0, \infty)$



For $t < 0$, no over lapping

$y(t) = 0$

For $t \geq 0$

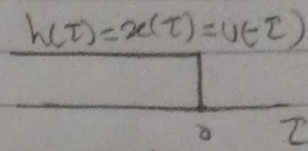


$$y(t) = \int_0^t 1 . d\tau = \tau \Big|_0^t = t$$

$y(t)$ has value only from 0 to ∞

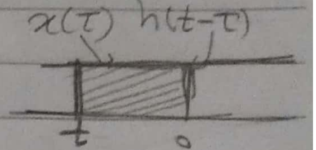
So $y(t) = t\,u(t) = r(t)$.

II $x(t) = u(t)$ $\qquad h(t) = u(-t)$



$y(t) = x(t) * h(t) \in (-\infty, 0]$

So The integration starts from $-\infty \Rightarrow 0$



for $t \leq 0$:

$$y(t) = \int_{-\infty} (t-\tau)\,d\tau = \begin{cases} 0, & t \geq 0 \\ \int_t^0 u(\tau - t) . d\tau, & t < 0 \end{cases} = \begin{cases} 0, & t \geq 0 \\ -t & t < 0 \end{cases}$$

$$= -t\,u(-t)$$

III $x(t) = u(t) - 2u(t-1) + u(t-2)$, $h(t) = u(t+1) - u(t-1)$

$$y(t) = [u(t) - 2u(t-1) + u(t-2)] * [u(t+1) - u(t-1)]$$

By using properties of convolution integral:-

$$y(t) = [u(t) * u(t+1)] - [u(t) * u(t-1)] - 2[u(t-1) * u(t+1)]$$

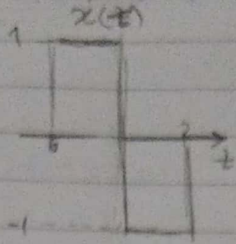
$$+ 2[u(t-1) * u(t-1)] + [u(t-2) * u(t+1)] - [u(t-2) * u(t-1)]$$

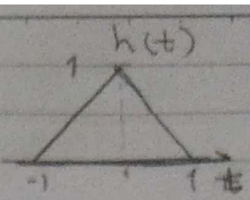
* Based on the results $u(t) * u(t) = r(t)$

$$y(t) = r(t+1) - r(t-1) - 2r(t) + 2r(t-2) + r(t-1) - r(t-3)$$

Q8:-

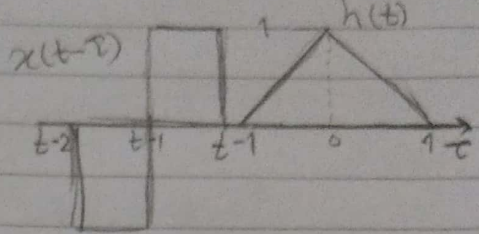
$y(t) \in [-1, 3]$





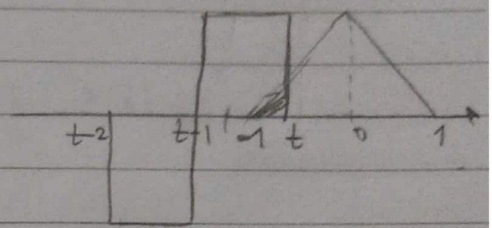
for $t < -1 \Rightarrow$ No overlapping

$y(t) = 0$



For $t \geq -1$, $t-1 \leq 1$

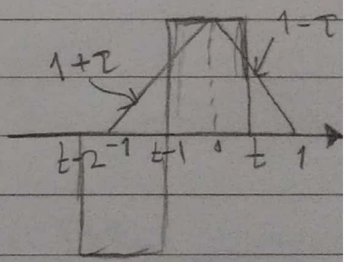
$-1 \leq t \leq 0$



$$y(t) = \int_{-1}^{t} (1+\tau)\,d\tau = \tau + \frac{\tau^2}{2} \Big|_{-1}^{t} = t + \frac{t^2}{2} + 1 + \frac{1}{2} = \frac{t^2}{2} + t + \frac{1}{2}$$

For $t < 1$, $t-1 \geq -1$

$0 \leq t < 1$



$$y(t) = \int_{-1}^{t-1} (-1)(1+\tau)\,d\tau + \int_{t-1}^{0} (1)(1+\tau)\,d\tau + \int_{0}^{t} (1)(1-\tau)\,d\tau$$

$$= -\tau - \frac{\tau^2}{2} \Big|_{-1}^{t-1} + \tau + \frac{\tau^2}{2} \Big|_{t-1}^{0} + \tau - \frac{\tau^2}{2} \Big|_{0}^{t}$$

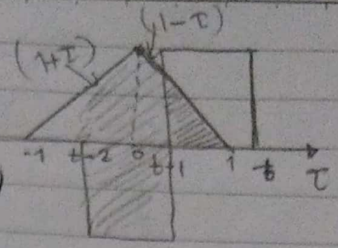
$$= -\left(t-1 + \frac{(t-1)^2}{2} + 1 - \frac{1}{2}\right) - \left(t-1 + \frac{(t-1)^2}{2}\right) + t - \frac{t^2}{2}$$

$$= -t + 1 - \frac{t^2 + 2t - 1}{2} - 1 + \frac{1}{2} - t + 1 - \frac{t^2 + 2t - 1}{2} + t - \frac{t^2}{2}$$

$$= \frac{\cancel{-2t} + \cancel{2t} - \cancel{2t} + \cancel{2t} + 2t}{2} + \frac{\cancel{2} - \cancel{1} - \cancel{2} + \cancel{1} + 2 - 1}{2} - \frac{t^2 - t^2 - t^2}{2}$$

$$= \frac{-3t^2}{2} + \frac{2t}{2} + \frac{1}{2} = \frac{-3t^2}{2} + t + \frac{1}{2}$$

For $t \geq 1$, $\quad t-1<1$
$1 \leq t < 2$



$$y(t) = \int_{t-2}^{0} (-1)(1+\tau)\, d\tau + \int_{0}^{t-1} (-1)(1-\tau)\, d\tau + \int_{t-1}^{1} (1)(1-\tau)\, d\tau$$

$$y(t) = -\left(\tau + \frac{\tau^2}{2}\right)\Big|_{t-2}^{0} + \left(\frac{\tau^2}{2} - \tau\right)\Big|_{0}^{t-1} + \tau - \frac{\tau^2}{2}\Big|_{t-1}^{1}$$

$$y(t) = (t-2) + \frac{(t-2)^2}{2} + \frac{(t-1)^2}{2} - t + 1 + 1 - \frac{\tau}{2} - t + 1 + \frac{(t-1)^2}{2}$$

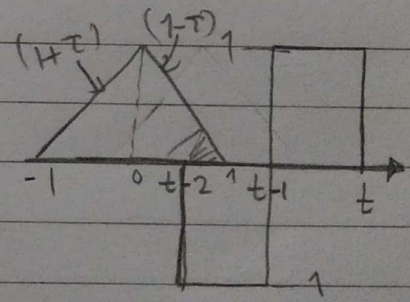
$$y(t) = t-2 \; \frac{+t^2+4t+4}{2} + \frac{t^2-2t+1}{2} - t+1+1-\frac{1}{2} - t+1 + \frac{t^2-2t+1}{2}$$

$$y(t) = \frac{\cancel{2t} - \cancel{4t} - 2t - \cancel{2t} - 2t - 2t}{2} \; \frac{\cancel{+4} - \cancel{1} + 1 + 2 + 2 - \cancel{1} + 2 + \cancel{1}}{2} + \frac{t^2+t^2+t^2}{2}$$

$$y(t) = \frac{3t^2}{2} - \frac{10t}{2} + \frac{7}{2}$$

$$y(t) = \frac{3t^2}{2} - 5t + \frac{7}{2}$$

For $t-2<1$, $\quad t-1 \geq 1$
$2 \leq t < 3$



$$y(t) = \int_{t-2}^{1} (-1)(1-\tau)\, d\tau = -\left(\tau - \frac{\tau^2}{2}\right)\Big|_{t-2}^{1}$$

$$= -\left(1 - \frac{1}{2} - t + 2 + \frac{(t-2)^2}{2}\right)$$

$$y(t) = \frac{-2+1-4+4}{2} + \frac{2t+4t}{2} - \frac{t^2}{2}$$

$$y(t) = \frac{-t^2}{2} + 3t - \frac{9}{2}$$

For $t>3 \Rightarrow y(t)=0 \Rightarrow$ no over lapping.

$$y(t)=\begin{cases} \frac{t^2}{2}+t+\frac{1}{2}, & -1\leq t<0 \\ -\frac{3t^2}{2}+t+\frac{1}{2}, & 0\leq t<1 \\ \frac{3t^2}{2}-5t+\frac{7}{2}, & 1\leq t<2 \\ -\frac{t^2}{2}+3t-\frac{9}{2}, & 2\leq t<3 \\ 0, & \text{elsewhere.} \end{cases}$$